



Review article

Single-phase convective heat transfer in microchannels: a review of experimental results

Gian Luca Morini

DIENCA, Università degli Studi di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy

Received 15 September 2003; received in revised form 3 January 2004; accepted 9 January 2004

Available online 27 March 2004

Abstract

A bibliographical review on the convective heat transfer through microchannels is presented. The available experimental works quoted in the open literature are critically analysed in order to highlight the main results obtained on the friction factor, on the laminar-to-turbulent transition and on the Nusselt number in channels having a hydraulic diameter less than 1 mm. A comparison of the experimental results quoted in the open literature is made. In many cases the experimental data of the friction factor and of the Nusselt number in microchannels disagree with the conventional theory but they also appear to be inconsistent with one another. Various reasons have been proposed to account for these differences. Rarefaction and compressibility effects, viscous dissipation effects, electro-osmotic effects (EDL), property variation effects, channel surface conditions (relative roughness) and experimental uncertainties have been invoked to explain the anomalous behaviour of the transport mechanisms through microchannels. By comparing the available experimental data on single-phase convective heat transfer through microchannels, it is evident that further systematic studies are required to generate a sufficient body of knowledge of the transport mechanism responsible for the variation of the flow structure and heat transfer in microchannels.

© 2004 Elsevier SAS. All rights reserved.

Keywords: Microchannels; Single-phase; Convective heat transfer; Friction factor; Laminar-to-turbulent transition; Nusselt number

1. Introduction

Micron size mechanical devices are today encountered both in commercial and in scientific applications. The development of micro-fluidics concerned with micro-systems has been particularly striking during the past 10 years.

As a rule of thumb all devices with characteristic dimensions between 1 μm and 1 mm are called micro-devices. Generally, the micro-systems can be subdivided into three categories:

- MEMS: Micro-Electro-Mechanical Systems (for instance, air bag acceleration sensors, HD reader, etc.).
- MOEMS: Micro-Opto-Electro-mechanical Systems (for instance, micro endoscope, etc.).
- MFD: Micro-flow devices (for instance, micro heat exchangers, micro-pumps, etc.).

In this paper only MFD devices will be analysed. Today, the research in this field is exploring different applications

which intimately involve the dynamics of fluids, the single phase and two-phase forced convective heat transfer and new potential applications are continuously being proposed.

It is possible to assert that the manufacture of MFD, like micro-pumps, micro-valves, micro-cold plates, micro-heat exchangers, and other micro-components and sensors used in chemical analysis, in biomedical diagnostics or in flow measurements, are today a consolidated reality.

Many technologies have been developed to realize miniaturized components. An interesting overview of the miniaturization technologies and their application to the energy systems has been presented by Ameel et al. [1] and, more recently, in [2]. A complete review of the research and development of micro-machined flow sensors is due to Nguyen [3]. The author underlines how, in the last years, the requirement of micro-flow sensors able to measure very small flow rates has increased. The results of this challenge have produced a new class of micro-machined flow sensors which have an integrated microchannel. Some interesting reviews of the works addressed in this topic are appearing in the open literature. Abramson and Tien [4] reviewed the recent progress in micro-scale engineering such as ther-

E-mail address: gianluca.morini@mail.ing.unibo.it (G.L. Morini).

Nomenclature

a, c	maximum and minimum channel width, respectively	m	Re	Reynolds number, = WD_h/ν	
b	channel depth	m	W_c	center-to-center distance between two adjacent microchannels	m
C	coefficient defined in Eq. (2), = fRe		<i>Greek symbols</i>		
$C_{f,l}$	coefficient defined in Eq. (1)		α	thermal diffusivity	$m^2 \cdot s^{-1}$
D_h	hydraulic diameter	m	ε	surface absolute roughness	μm
f	Fanning friction factor		γ	channel aspect ratio, = b/a	
L	length of microchannel	m	λ	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
Nu	Nusselt number, = hD_h/λ		ν	kinetic viscosity	$m^2 \cdot s^{-1}$
Pr	Prandtl number, = ν/α				

mal transport phenomena, experimental and computational techniques, thermal micro-devices and laser applications. The authors underlined the more promising research lines in these areas.

It is evident that the understanding of the microscale transport phenomena is very important for the designer of MFD. For this reason, many studies have been conducted in order to analyse the behaviour of the convective flow through microchannels. Reviews of these works have been presented by many authors. Duncan and Peterson [5] provided a wide review of microscale convective, conduction and radiation heat transfer. Peng and Wang [6] gave a review of their own extensive research on the one-phase and two-phase microscale convective heat transfer. Bailey et al. [7] concentrated their attention on the single phase forced convection through microchannels and concluded that the literature is inconclusive with respect to the effect of miniaturization on heat transfer and pressure drop. In these review works it is highlighted that, in many cases, the experimental results obtained for laminar flow through microchannels present a significant deviation from the predictions of the conventional theory. Recently, Palm [8], Sobhan and Garimella [9] and Obot [10] presented a critical review of the published results on the friction factor and convective heat transfer in microchannels. The work by Obot [10] is particularly interesting because it represents a first attempt to compare critically, also from a quantitative point of view, the experimental results obtained for microchannels with the conventional theory validated for large sized channels. Obot demonstrated that the deviations from the conventional theory of the pressure drop and the convective heat transfer data are, in some cases, fictitious.

Also Bowman and Maynes [11] presented a complete review of the literature in the area of micro-heat exchangers focusing primarily on the works published between the early 1990s and 2001.

A more complete and recent analysis of the literature on the behaviour of liquid flow [12] and gas flow [13] through microchannels is due to Rostami et al.

Kandlikar and Grande [14] reviewed the advancement in heat transfer technology from historical perspective.

The use of microchannels in high flux cooling application is discussed with single-phase and two-phase flows. The current state of fabrication technology of microchannels is well reviewed.

The geometry of MFD microchannels depends on the technology used to build the MFD. As mentioned above, there are many techniques to realize miniaturized channels but four process technologies are imposing for system miniaturization:

- Micromechanical machining (such as diamond machining, laser processes, focused ion beam, microdrilling);
- X-ray micromachining (such as LIGA Litographie-Galvanoformung-Abformung);
- Photolithographic-based processes (such as Si chemical etching);
- Surface and surface-proximity-micromachining (epimicromachining) processing techniques.

The photolithographic processes are particularly indicated for silicon wafers; this technology, born in the electronic field, is fairly mature. The cross-sectional shapes that can be obtained for microchannels realized with this technique are limited. In fact, the microchannels produced by chemical etching directly on the silicon wafers have a cross-sectional shape that depends on a variety of factors such as the crystallographic nature of the silicon used. When a photo-lithographic based process is employed, one can obtain microchannels having a cross-section fixed by the orientation of the silicon crystal planes; for example, the microchannels etched in $\langle 100 \rangle$ or in $\langle 110 \rangle$ silicon by using a KOH solution have a trapezoidal cross-section (with an apex angle of 54.74° imposed by the crystallographic morphology of the $\langle 100 \rangle$ silicon) or a rectangular cross-section, respectively. Since this technique is very diffuse, in literature there are many experimental results obtained for microchannels having a trapezoidal, rectangular and double-trapezoidal (hexagonal) cross-section (see Fig. 1).

On the contrary, with the other techniques it is possible to realize miniaturized channels having, theoretically, any cross-section.

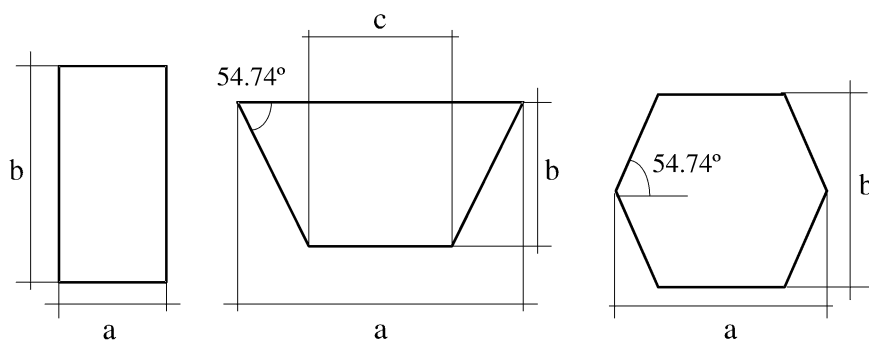


Fig. 1. Rectangular, trapezoidal and double-trapezoidal KOH-etched microchannels.

The flow regime through a MFD depends strongly on the method used in order to induce the motion of the fluid. The fluid flow in a MFD can be obtained in two different way:

- By applying an external pressure gradient (*pressure-driven motion*). In this case a Poiseuille flow profile is generated along the channel. In fact, because the Reynolds number associated to the flow is in general small (due to the small hydraulic diameter of the channel), the flow is usually laminar and the velocity varies across the entire cross-sectional area of the channel.
- By applying an external electric field (*electrokinetically-driven flow*). In this case the fluid velocity only varies within the so-called Debye screening layer near the channel walls. Whitesides and Stroock [15] showed the velocity profile for an electrokinetically driven flow in a microtube by using a flow visualisation technique. It is experimentally demonstrated that, in this case, the profile is practically uniform (slug flow) across the entire cross-section.

In this work the main experimental results on the convective flow through microchannels will be analysed with regard to the prediction of the friction factors, the laminar-to-turbulent transition and the Nusselt numbers. The experimental results will be compared with the prediction of the conventional theory for continuum flows in order to underline the discrepancies between the theory and the experimental results for microchannels.

2. Convective flow through microchannels: State-of-the-art

In this section the main experimental results on the laminar flows through microchannels which have appeared in the open literature in the last decade are quoted and commented.

2.1. Friction factor

The first work in which the friction factor through silicon microchannels was measured is due to Wu and Little [16]. The microchannels tested were made in glass and in (100) silicon substrate by using a photo-litographic technique and sealed with pyrex by using anodic bonding. Different microchannels were tested, having a trapezoidal cross-section with a hydraulic diameter equal to 55.81, 55.92 and 72.38 μm . Gases were used as test fluids (N_2 , H_2 , Ar); the measured values of the friction factor were larger (10–30%) than those predicted by the conventional theory. The authors concluded that the deviations are due to the great relative roughness and to the asymmetric distribution of the surface roughness on the microchannel walls. Acosta et al. [17] presented friction factors for rectangular microchannels with a hydraulic diameter ranging between 368.9 and 990.4 μm . The rectangular channels investigated are characterised by a very small value of the aspect ratio ($0.019 < a/b < 0.05$). The tested microchannels showed very similar values of the friction factor; this fact is consistent with the conventional asymptotic behaviour of the Poiseuille number (fRe) for channels with small aspect ratios.

Harley and Bau [18] measured the friction factor for a trapezoidal silicon microchannel with a hydraulic diameter of 45 μm and for a rectangular microchannel having a hydraulic diameter of 67 μm . By using isopropanol as test fluid, they found that the experimental Poiseuille number is larger than the expected theoretical value.

An opposite trend is found in the succeeding works by Pfaller et al. [19–21]; they tested rectangular microchannels having a very small hydraulic diameter (1.6 and 3.4 μm) and trapezoidal microchannels, nominally 100 μm wide and 0.5–50 μm deep, for Reynolds numbers from 50 to 300. The fluids used in the test were liquids (isopropanol, silicon oil) and gases (helium and nitrogen). The measured friction factor was consistently lower than the theoretical predictions. The Poiseuille number decreased with the Reynolds number in the smallest channel studied. They also showed that the viscosity becomes size-dependent at micro-scales, as the thermal conductivity does in a very thin solid layer. They found that the polar nature of the fluid affects the friction factor.

Choi et al. [22] measured the friction factor for a fully developed laminar flow of nitrogen gas through silica micropipes having a diameter of 3, 7, 10, 53, 81 μm with Reynolds numbers ranging between 30 and 20000. They found that the Poiseuille number (fRe) in laminar regime is lower than the conventional value ($fRe = 16$). No variations of the friction factor with the wall roughness is evidenced.

The effect of the temperature variation on the Poiseuille number was studied by Urbanek et al. [23]. In this experiment, three isomers of alcohol (isopropanol) were tested in triangular and trapezoidal microchannels etched on a silicon (100) substrate, having a hydraulic diameter between 5 and 25 μm . The friction factors showed higher values with respect to the predictions of the conventional theory.

Rahman and Gui [24,25] measured experimentally the friction factors for water through trapezoidal microchannels having a width of 1 mm and with channel depths ranging between 79 and 325 μm . They presented experimental values of the friction factor and the Nusselt number. Their experimental friction factors agreed with the conventional theory. Wilding et al. [26] analysed a laminar flow of water and of various biological fluids in glass-capped silicon microchannels having a trapezoidal cross-section. The channels tested had dimensions ranging between 40 and 150 μm in width and between 20 and 40 μm in height. The data indicated an increase of the friction factor with respect to the theoretical value.

Arklic et al. [27] studied the flow of helium through a deep rectangular silicon microchannel ($D_h = 2.6 \mu\text{m}$). The authors revealed lower friction factors compared with the conventional results for large channels; they demonstrated that the mass flow rate through the microchannel can be accurately predicted by using a slip flow boundary condition at the walls.

Pong et al. [28] studied experimentally a flow of nitrogen and helium in rectangular microchannels with a depth of 1.2 μm and a width of 5–40 μm . They measured the axial pressure distribution in the channel long 4500 μm ; the pressure distribution along the microchannel was found not linear. They observed that this fact can be due to the rarefaction and the compressibility effects.

Liu et al. [29] analysed the pressure drop through rectangular microchannels by using helium as testing fluid. They used the same test rig described by Pong et al. [28]. They found that the pressure drop was smaller than that expected. The flow rate was very accurately predicted by means of a slip boundary condition with an accommodation factor equal to 1.

Shih et al. [30] completed the measurements of Liu et al. [29] by using nitrogen and helium as working fluids. The slip flow model predicted correctly the pressure drop for an inlet pressure lower than 0.25 MPa with different accommodation factors for helium and nitrogen; on the contrary, for an inlet pressure greater than 0.25 MPa the slip flow model underpredicted the experimental data.

Peng et al. [31,32] considered a water flow through rectangular machined steel grooves. The hydraulic diameters investigated ranged between 133 and 367 μm . For Reynolds numbers ranging between 8 and 800, they evidenced higher friction factors with respect to the macroscale predictions; the main result obtained in these works is that the friction factor and the Reynolds number do not seem to be inversely proportional, as indicated by the conventional theory for laminar flow. The authors proposed empirical correlations in order to calculate the friction factor in laminar and in turbulent regime.

Yu et al. [33] analysed experimentally a nitrogen and water flow through microtubes in silica having a diameter of 19, 52 and 102 μm for Reynolds numbers ranging between 250 and 20000. They observed friction factors lower than theoretical macroscale predictions. They analysed the role of the relative roughness of the tube.

An experimental study of gas flow (nitrogen, helium and argon) in silicon (100) microchannels having a trapezoidal and rectangular cross-section was conducted by Harley et al. [34]. The hydraulic diameter of the microchannels tested ranged between 1.01 and 35.91 μm and the aspect ratio varied between 0.0053 and 0.161 (for the trapezoidal geometry the aspect ratio height/width is based on the maximum width). In this work, the compressibility effects and the rarefaction effects on the Poiseuille number were investigated. They observed smaller friction factor with respect to the predictions of the conventional theory and they explained this trend by using an isothermal “locally fully developed” first-order slip-flow model.

A further analysis of the experimental data of Rahman and Gui [24] was conducted by Gui and Scaringe [35]; they proved that the friction factors agree well with theoretical solution in the laminar regime.

Jiang et al. [36] studied the behaviour of a water flow through rectangular and trapezoidal cross-section channels. The microchannels used in this study were etched in a silicon substrate and capped with a glass wafer. The channel dimensions ranged from 35 to 110 μm in width and from 13.4 to 46 μm in height. The experimental data revealed that the friction factor is greater than the expected theoretical value. It is interesting to note that in a later work Jiang et al. [37] studied a water flow through trapezoidal microchannels with a hydraulic diameter ranging from 35 to 120 μm and Reynolds numbers from 1 to 30 and their measurements revealed friction factors lower than the conventional theory.

Richter et al. [38] performed flow rate measurements for water through microchannels with various geometries for liquid dosing applications (flow rates between 0.01 and 1000 $\mu\text{l}\cdot\text{min}^{-1}$). The channels were micromachined and anisotropically etched in a (100) silicon wafer. The cross-section was trapezoidal and the maximum width of the channels ranged between 28 and 182 μm . A linear relation between the flow rate and the pressure drop was noticed. The authors observed that the conventional theory can predict the

experimental results very accurately taking into account the variation of the fluid viscosity with temperature.

Experimental investigations on the Poiseuille number for deionized water through deep rectangular microchannels ($D_h = 404 \mu\text{m}$) made in a $\langle 110 \rangle$ silicon substrate were conducted by Harms et al. [39,40]. The friction factor for a microchannel having a hydraulic diameter of $404 \mu\text{m}$ [40] was reasonably well predicted by the conventional theory in the laminar regime.

Webb and Zhang [41] studied channels of rectangular shape with hydraulic diameters from $960 \mu\text{m}$ to 2.13 mm). They observed that the classical correlations were able to predict the single-phase heat transfer coefficient and the friction factor for multiport rectangular-cross-section channels for the hydraulic diameters investigated.

Pfund et al. [42] measured the pressure drop of water flowing along rectangular microchannels having a hydraulic diameter ranging from 200 to $900 \mu\text{m}$ and Reynolds numbers between 40 and 4000 . In the laminar regime their data showed a good agreement with the conventional theory. They observed that in the laminar regime the friction factor increases with the surface roughness of the walls.

An experimental and numerical work due to Flockhart and Dhariwal [43] deals with the flow characteristics in trapezoidal channels etched in $\langle 100 \rangle$ silicon with a hydraulic diameter ranging from 50 to $120 \mu\text{m}$. Distilled water was used in the investigation; the flow regime was laminar. A good agreement with the theoretical predictions on the friction factor in large channels was found.

Mala and Li [44] studied experimentally a flow of water through circular microtubes of fused silica and stainless steel with a hydraulic diameter ranging between $50 \mu\text{m}$ and $254 \mu\text{m}$. For a smaller diameter, they observed a nonlinear trend between pressure drop and flow rate for low Reynolds numbers, and that the friction factors were consistently higher with respect to the conventional values. In addition, their experimental results indicated material dependence of the flow behaviour; for the same flow rate and diameter, a higher pressure gradient is required in a fused silica microtube than in a stainless steel microtube. Experimental data on the pressure drop of water in the laminar regime through rectangular microchannels were obtained by Papautsky et al. [45]. The tested microchannels were rectangular with a width ranging between 50 and $600 \mu\text{m}$ and a height between 20 and $30 \mu\text{m}$. Their experimental friction factors were higher than the conventional ones. Using PIV measurements Meinhart et al. [46] analysed the laminar flow through a glass rectangular microchannel with a height of $30 \mu\text{m}$ and a width of $300 \mu\text{m}$. The experimental results agreed well with the analytical velocity profile for Newtonian flow in rectangular channels. To the best of author's knowledge, this work is the only experimental work in which measurements of the local velocity distribution across test sections are made. Xu et al. [47] performed experimental investigations on water flow in microchannels with hydraulic diameters ranging from 50 to $300 \mu\text{m}$.

They observed that the flow characteristics deviated from conventional theory for channel dimensions below $100 \mu\text{m}$. The friction factor was smaller than that predicted by the Hagen–Poiseuille law.

Qu et al. [48] conducted an experimental investigation on the flow characteristics of water through trapezoidal silicon microchannels with a hydraulic diameter ranging between 51 and $169 \mu\text{m}$. The flow rate and the pressure drop across the microchannels were measured and the fully developed friction factors were calculated. The calculated friction factors are higher than the expected values obtained using the conventional theory for laminar flow. The authors proposed a roughness-viscosity model to explain the experimental data.

Sharp et al. [49] observed a good agreement between the measured friction factors in circular microtubes and the conventional theory. They considered water flow through micropipes having a hydraulic diameter ranging between 75 and $242 \mu\text{m}$ for Reynolds numbers in the range 50 – 2500 .

Very interesting experimental work was conducted by Xu et al. [50]. They compared the experimental data obtained with rectangular microchannels machined in an aluminium plate and bonded with a plexiglass plate with the data obtained by using microchannels etched on a silicon wafer and bonded with a pyrex cover. The hydraulic diameters investigated ranged from 46.8 to $344.3 \mu\text{m}$ for aluminium microchannels and from 29.59 to $79.08 \mu\text{m}$ for silicon microchannels. They concluded that for liquid flow through microchannels with a hydraulic diameter greater than $30 \mu\text{m}$ the conventional results obtained by using the Navier–Stokes equation for an incompressible, Newtonian fluid in the laminar regime agree very well with the experimental data.

Ding et al. [51] performed experimental work to investigate the pressure drop for R134a and R12 through stainless steel microchannels with a triangular and a rectangular cross-section having a hydraulic diameter of 400 and $600 \mu\text{m}$. The friction factor was higher than the conventional macroscale prediction. Their experimental results showed that the influence of the wall roughness on the friction factor was strong. In particular, they found that the Poiseuille number in the laminar regime is not constant for a fixed geometry of the microchannel cross-section but depends on the Reynolds number. They proposed a very similar correlation for the friction factor in the laminar regime to the correlation proposed by Peng et al. [31,32].

Araki et al. [52] investigated frictional characteristics of nitrogen and helium flows through three different trapezoidal microchannels having a hydraulic diameter ranging from 3 to $10 \mu\text{m}$. The measured friction factor was smaller than that predicted by the conventional theory. They explained this deviation by observing that the rarefaction effects could be significant.

Celata et al. [53] performed an experimental analysis of the friction factor in a capillary tube having a diameter of $130 \mu\text{m}$ by using R114 as test fluid. They observed the behavior of the laminar-to-turbulent transition for Reynolds numbers ranging between 100 and 8000 . Experiments indi-

cate that in the laminar flow regime the friction factor is in good agreement with the conventional theory.

Judy et al. [54] measured the friction losses of water, hexane and isopropanol flowing in fused silica capillaries in order to study the effect of the fluid polarity. The capillary diameters ranged from 20 to 150 μm . For diameters lower than 100 μm the friction factor deviated from the conventional theory. The friction factor was lower than expected and the deviations were higher for decreasing diameters.

Li et al. [55] investigated the friction factor of a nitrogen flow through five microtubes with diameters ranging from 80 to 166.6 μm . They found that the pressure drop along the tube became nonlinear at Mach numbers higher than 0.3. In this case, the friction factor is higher than the prediction of the conventional theory.

Li et al. [56] used glass, silicon and stainless steel microtubes with diameters ranging from 79.9 to 166.3 μm , from 100.25 to 205.3 μm , from 128.76 to 179.8 μm , respectively, in order to measure the friction factors for deionized water. They observed that the glass and the silicon microtubes can be considered smooth; on the contrary, the stainless steel microtubes exhibited a relative roughness of 3–4%. They concluded that for glass and silicon microtubes the conventional theory in the laminar regime holds. For stainless steel microtubes the friction factors were higher than the prediction of the classical theory. They concluded that the relative roughness cannot be neglected for microtubes also in the laminar regime.

Yang et al. [57] provided a test of the friction characteristic of air, water and liquid refrigerant R134a through ten tubes with a diameter from 173 to 4010 μm . The test results evidenced that the friction factor for water and refrigerant R134a agree very well with the conventional theory in the laminar and in the turbulent regime. On the contrary, for air flow in the turbulent regime the measured friction factors were significantly lower than those predicted by the conventional theory.

Pfund et al. [58] measured the friction factor for water flowing in high aspect ratio smooth and rough rectangular channels with depths ranging from 128 to 521 μm for Reynolds numbers between 60 and 3450. They found that the friction factor in laminar flow were significantly greater than the classical value, in particular when the roughened channel ($\varepsilon/D_h = 3\%$) was tested.

Debray et al. [59] presented measurements of the friction factor in a flat rectangular microchannel having a hydraulic diameter of 590 μm for Reynolds numbers ranging between 70 and 6300. Their results confirmed the prediction of the conventional theory.

Lalonde et al. [60] presented experimental data on the friction factor for air flowing through a microtube with a diameter of 52.8 μm . The results indicated a good agreement with the predictions of the conventional theory. Turner et al. [61] presented an experimental investigation on compressible gas flow through rough and smooth rectangular

microchannels with hydraulic diameters from 4 to 100 μm . Nitrogen, helium and air were used as test fluids. All the measurements were made in the laminar regime with Reynolds numbers ranging from 0.02 to 1000. The investigation showed that the friction factor for both smooth and rough microchannels was in agreement with the conventional theory.

Jiang et al. [62] tested a micro-heat exchanger with microchannels made from 0.7 mm thick pure copper plates on a wire cutting machine. The channels were rectangular and had a hydraulic diameter of 300 μm . Measurements with an electron microscope showed that the surface roughness of the microchannels was between 5.8 and 36.3 μm ; this means an average relative roughness of the microchannels of 0.1. In addition, due to the microfabrication technique used, the cross-section of the microchannels is not perfectly rectangular. The measured friction factors were larger than the values predicted by the conventional theory. They explained this fact by observing that the effect of the hydrodynamic entrance region was not negligible for the short microchannels tested.

Ren et al. [63] verified experimentally the role of the interfacial electrokinetic effects on liquid flow in microchannels. They measured the magnitude of the additional flow resistance caused by the electrokinetic effect through three silicon rectangular microchannels having a hydraulic diameter of 28.1, 56.1 and 80.3 μm . De-ionized ultra-filtered (DIUF) water and aqueous KCl solutions of two different concentrations (10^{-4} and 10^{-2} $\text{kmol}\cdot\text{m}^{-3}$) are used as working fluids. The measured pressure drops for the pure water and the low concentration solution were found to be significantly higher than the pressure drop for the liquid with high concentration of KCl, at the same Reynolds number. The difference tends to increase for small hydraulic diameters. In particular, any difference on the pressure drop was evidenced for the microchannel with a hydraulic diameter of 80.3 μm . On the contrary, the maximum difference was experienced for the smaller microchannel (20% higher flow resistance). For this microchannel DIUF water evidenced higher flow resistance. The authors concluded that these results confirmed that the electrical-double layer effect plays an important role on the pressure drop through small microchannels.

Kandlikar et al. [64] investigated experimentally the role of the relative roughness on the pressure drop in two microtubes with different diameters (1067 and 620 μm). They found that for the 1067 μm diameter tube, the effects on pressure drop of the variation of the relative roughness from $\varepsilon/D = 0.00178$ to 0.003 are insignificant and the tube can be considered smooth. However, for the 620 μm tube, the same relative roughness value increases the pressure drop; in other words, a 620 μm tube with a relative roughness $\varepsilon/D = 0.003$ cannot be considered as smooth. This fact suggests that the relative roughness could play a more important role in microchannels than in conventional sized channels (sizing effects).

Gao et al. [65] presented an experimental investigation of the frictional characteristics of demineralised water through rectangular microchannels. The facility presented was designed to modify easily the channel height from 1000 to 100 μm . They found that the experimental Poiseuille number was in agreement with the prediction of the conventional theory for all the hydraulic diameters considered (D_h ranged between 199.2 and 1923 μm).

Warrier et al. [66] presented experimental results on heat transfer and pressure drop for an aluminium rectangular microchannel having a hydraulic diameter of 750 μm by using FC-84 as test fluid. The experimental data relating to the pressure drop for the laminar flow were in good agreement with the conventional predictions. Judy et al. [67] investigated a pressure driven flow through round and square microchannels in fused silica and stainless steel with diameters in the range of 47–101 μm and for Reynolds numbers ranging between 8 and 2300. Distilled water, methanol and isopropanol were used in order to study the effect of the polarity of the fluid on the friction factor. The data on the pressure drop revealed no distinguishable deviation from the Stokes flow theory.

Hegab et al. [68] studied experimentally the fluid flow and the heat transfer of R134a in rectangular microchannels micromilled in aluminium with hydraulic diameters ranging from 112 to 210 μm and for an aspect ratio from 1 to 0.667. The few data collected in the laminar regime were very close to the predictions using the classical laminar flow theory. The experimental results in the transition region and those obtained in the turbulent regime indicated that the friction factor was lower than the values predicted by the conventional theory. The relative roughness ranged between 0.0016 and 0.0089.

Qu and Mudawar [69] measured the pressure drop of water flowing in a single-phase microchannel heat sink. They found that the pressure drop was in good agreement with the predicted values of the conventional theory for continuum flows. The change in the slope of the pressure drop with Reynolds number was attributed to the temperature dependence of water viscosity.

Bucci et al. [70] measured the friction factor for water flowing in a 290 μm pipe characterized by a relative roughness of 0.75%. They observed that the friction factor follows the Hagen–Poiseuille law up to a Reynolds number equal to 1500.

Wu and Cheng [71] conducted an experimental analysis in order to quantify the friction factor of laminar flow of deionized water in smooth silicon microchannels of trapezoidal cross-section with hydraulic diameters in the range of 25.9 and 291 μm . Their experimental data are found to be in good agreement with the predictions of the conventional theory. The authors concluded that the Navier–Stokes equations are still valid for the laminar flow of deionized water in smooth silicon microchannels having hydraulic diameters as small as 26 μm .

Li et al. [72] used glass, silicon and stainless steel microtubes with diameters of 79.9–166.3 μm in order to test the laminar frictional resistance of a deionized water flow. They concluded that the Poiseuille number remains approximately equal to 16 through smooth the glass and silicon microtubes. On the contrary, the Poiseuille number was found 15–37% higher than 16 in the rough stainless steel microtubes ($\varepsilon/D_h = 3\text{--}4\%$). The authors highlighted that this conclusion is in contradiction with the conventional theory in which the effect of internal wall relative roughness on laminar flow characteristics are ignored for relative roughness lower than 5%.

In Table 1 are compared the experimental results on the friction factor for liquid laminar flow through silicon microchannels quoted in literature. It is interesting to note that, if the experimental results are used in order to establish the validity of the conventional theory for microchannels, the answer obtained is not univocal. In fact some authors found that the predictions of the conventional theory agree with the experimental results on the friction factor; on the other hand, for the same range of hydraulic diameter, someone found the opposite result. It is important to underline that, in some cases, the authors compared their experimental Poiseuille numbers by using the conventional results for pipes ($fRe = 16$) even if the microchannel cross-section was rectangular or trapezoidal.

It is possible to sum up the main results quoted in the open literature on the friction factor in MFD highlighting the peculiarities, proposed by different authors, with respect to the conventional macrochannels:

- the friction factor for laminar fully developed flow is found to be lower than the conventional value [19–22, 27–30, 33, 34, 37, 47, 52, 54, 57]; this result has been obtained in particular for gas flow through microchannels;
- the friction factor for laminar fully developed flow is found to be higher than the conventional value [16, 18, 23, 26, 31, 32, 36, 44, 45, 48, 51, 55, 58, 62–64, 72];
- the Poiseuille number for laminar fully developed flow depends on the Reynolds number [19–21, 31, 32, 51];
- the friction factor for gaseous laminar fully developed flow decreases with the Knudsen number [27–30, 34, 52];
- the friction factor for gaseous laminar fully developed flow with Mach numbers greater than 0.3 augments with the Mach number [28, 34, 55] due to the effect of gas compressibility;
- the friction factor depends on the material of the microchannel walls (metals, semi-conductors and so on) and/or on the test fluid (polar fluid or not), thus evidencing the importance of electro-osmotic phenomena at microscales [19–21, 44, 54, 67];
- the friction factor depends on the relative roughness of the walls of the microchannels also in laminar regime [16, 33, 42, 48, 51, 53, 56, 58, 62, 64, 70, 72].

Table 1
Experimental results on the Poiseuille number for laminar flows in microchannel: Liquids

	fRe	D_h [μm]	Cross-section	Test fluids
Harley and Bau [18]	$\uparrow\uparrow$	45–67	Rect.-Trap.	Isopropanol
Pfalther et al. [19–21]	$\downarrow\downarrow$	1.6–65	Rect.-Trap.	Isopropanol, silicon oil
Urbanek et al. [23]	$\uparrow\uparrow$	5–25	Trap.	Isopropanol
Rahman and Gui [24,25]	\approx	79–325	Trap.	Water
Gui and Scaringe [35]				
Wilding et al. [26]	$\uparrow\uparrow$	26–63	Trap.	Biological fluids, water
Peng et al. [31,32]	$\uparrow\uparrow$	133–367	Rect.	Water
Yu et al. [33]	$\downarrow\downarrow$	19–102	Circ.	Water
Jiang et al. [36]	$\uparrow\uparrow$	20–65	Rect.-Trap.	Water
Jiang et al. [37]	$\downarrow\downarrow$	35–120	Trap.	Water
Richter et al. [38]	\approx	18–116	Trap.	Water
Harms et al. [39,40]	\approx	404–1923	Rect.	Water
Webb and Zhang [41]	\approx	960–2000	Rect.	R134a
Pfund et al. [42]	\approx	200–900	Rect.	Water
Flockhart and Dhariwal [43]	\approx	50–120	Trap.	Water
Mala and Li [44]	$\uparrow\uparrow$	50–254	Circ.	Water
Papautsky et al. [45]	$\uparrow\uparrow$	50–600	Rect.	Water
Meinhart et al. [46]	\approx	54.5	Rect.	Water
Xu et al. [47]	$\downarrow\downarrow$	50–300	Rect.	Water
Qu et al. [48]	$\uparrow\uparrow$	51–169	Trap.	Water
Sharp et al. [49]	\approx	75–242	Circ.	Water
Xu et al. [50]	\approx	29.59–344.3	Rect.	Water
Ding et al. [51]	$\uparrow\uparrow$	400–600	Rect.-Tri.	R134a
Celata et al. [53]	\approx	130	Circ.	R114
Judy et al. [54]	$\downarrow\downarrow$	20–150	Circ.	Isopropanol, water, methanol
Li et al. [56]	\approx	79.9–205.3	Circ.	Water
Yang et al. [57]	\approx	173–4010	Circ.	Water, R134a
Pfund et al. [58]	$\uparrow\uparrow$	128–521	Rect.	Water
Debray et al. [59]	\approx	590–2218	Rect.	Water
Jiang et al. [62]	$\uparrow\uparrow$	300	Rect.	Water
Ren et al. [63]	$\uparrow\uparrow$	28.1–80.3	Rect.	DIUF water, aqueous KCl solutions
Kandlikar et al. [64]	$\uparrow\uparrow$	620–1067	Circ.	Water
Gao et al. [65]	\approx	199.2–1923	Rect.	Water
Warrier et al. [66]	\approx	750	Rect.	FC-84
Judy et al. [67]	\approx	47–101	Rect.	Isopropanol, water, methanol
Hegab et al. [68]	\approx	112–210	Rect.	R134a
Qu and Mudawar [69]	\approx	349	Rect.	Water
Bucci et al. [70]	\approx	290	Circ.	Water
Wu and Cheng [71]	\approx	25.9–291	Trap.	Water
Li et al. [72]	$\uparrow\uparrow$	79.9–166.3	Circ.	Water

$\uparrow\uparrow$ — fRe higher than the conventional theory.

$\downarrow\downarrow$ — fRe lower than the conventional theory.

\approx — fRe agrees with the conventional theory.

By observing the data quoted in Table 2, it is possible to note that in general for gases the friction factor is found to be smaller than the value of the conventional theory for continuum flows; for liquids, the opposite result is found.

2.2. Laminar-to-turbulent transition

In the experimental work of Wu and Little [16] some silicon and glass microchannels were tested. The glass channels had a rectangular cross-section with two rounded corners and with a high value of relative roughness ($\varepsilon/D = 0.2$ – 0.3) displaced non-uniformly along the wetted perimeter. The silicon microchannels were realized with a photolithographic technique on a (100) silicon wafer; in this case the channels were trapezoidal and could be considered smooth. Compar-

ing the results of Wu and Little [16] for glass and silicon channels, the role of the relative roughness on the transition was evidenced. Wu and Little concluded that the transition occurs at Reynolds numbers ranging from 1000 to 3000.

Acosta et al. [17] presented an analysis of the friction factors for isothermal gas flows in rectangular microchannels; the investigated rectangular channels have a very small value of the aspect ratio ($0.019 < \gamma < 0.05$). The microchannels tested evidenced very similar trends with respect to the laminar-to-turbulent transition; the critical Reynolds number is about 2770, as quoted by Obot [10].

The experimental data of Choi et al. [22] suggested that the critical Reynolds number decreases with the hydraulic diameter; in particular, the transition occurred at Reynolds

Table 2
Experimental results on the Poiseuille number for laminar flows in microchannel: gases

	fRe	D_h [μm]	Cross-section	Test fluids
Wu and Little [16]	$\uparrow\uparrow$	55.8–72.4	Trap.	N_2 , H_2 , Ar
Acosta et al. [17]	\approx	368.9–990.4	Rect.	He
Pfalher et al. [19–21]	$\downarrow\downarrow$	1.6–65	Rect.-Trap.	N_2 , He
Choi et al. [22]	$\downarrow\downarrow$	3–81	Circ.	N_2
Arklic et al. [27]	$\downarrow\downarrow$	2.6	Rect.	He
Pong et al. [28]	$\downarrow\downarrow$	1.94–2.33	Rect.	N_2 , He
Liu et al. [29]	$\downarrow\downarrow$	2.33	Rect.	N_2 , He
Shih et al. [30]	$\downarrow\downarrow$	2.33	Rect.	N_2 , He
Yu et al. [33]	$\downarrow\downarrow$	19–102	Circ.	N_2
Harley et al. [34]	$\downarrow\downarrow$	1.01–35.91	Rect.-Trap.	N_2 , He, Ar
Araki et al. [52]	$\downarrow\downarrow$	3–10	Trap.	N_2 , He
Li et al. [55]	$\uparrow\uparrow$	128.8–179.8	Circ.	N_2
Yang et al. [57]	$\downarrow\downarrow$	173–4010	Circ.	Air
Lalonde et al. [60]	\approx	52.8	Circ.	Air
Turner et al. [61]	\approx	4–100	Rect.	Air, N_2 , He

$\uparrow\uparrow$ — fRe higher than the conventional theory.

$\downarrow\downarrow$ — fRe lower than the conventional theory.

\approx — fRe agrees with the conventional theory.

equal to 2000 for a circular tube with a hydraulic diameter of 53 μm and at 500 for a hydraulic diameter of 9.7 μm .

Wang and Peng [73], analysed the single-phase forced flow convection of water or methanol through rectangular microchannels machined in parallel on a stainless steel plate. In this experiment, the laminar-to-turbulent flow transition of a non-isothermal flow is studied in terms of critical Reynolds numbers. In this case, the variations in the liquid thermo-physical properties due to the increase in the temperature along the channels played an important role. The transition region is deduced by observing the trend of the average Nusselt number. They found that the transition from laminar to turbulent regime occurs for Reynolds numbers ranging between 300 and 800. It can be underlined that in this work there is no information about the roughness of the walls of the tested microchannels. A further analysis was conducted by Peng and Peterson [74] using the same test sections analysed in [73]. The effect of the variation of the fluid properties with the temperature for forced convection of water and methanol through metallic rectangular microchannels was investigated. They observed the laminar region for Reynolds numbers lower than 400. Also in this case, the transition region was deduced by observing the trend of the average Nusselt number. The authors underlined that the heat transfer behaviour in the laminar and transition regions was quite unusual and complicated. The authors evidenced that for a water flow the Reynolds number could double over the length of the microchannels. They used the Reynolds number at the inlet for their considerations. No information on the relative roughness of the tested microchannels is given.

Peng and Peterson [75] quoted the transition Reynolds numbers for different aspect ratios of rectangular microchannels for water and methanol flows. The microchannels tested were metallic and they were obtained in a very similar way

to the microchannels tested by Jiang et al. [62]. Also in this case, the fluid was heated along the channel. With respect to the prediction of the conventional theory an early transition was observed; it was found that the transition region, in terms of Reynolds numbers, changed with the aspect ratio of the channel. The transition Reynolds number and transition range diminished with reductions in the microchannel dimensions.

Yu et al. [33] found, for a circular tube, that the transition occurred for Reynolds numbers from 1700 to 6000, in line with the conventional theory for continuum flows. Gui and Scaringe [35], in their experience, found a good agreement with the conventional theory for the friction factors of water through trapezoidal microchannels. The roughness of the channels was not measured but they suggested a value of ε/D_h equal to 0.015. The critical Reynolds number found experimentally was 1400 for the microchannels with the aspect ratio (γ) equal to 0.079.

Nguyen et al. [76] investigated the flow friction and the forced convection through trapezoidal microchannels made by using an anisotropic etching in silicon. The cross-section was 500 μm deep and 1707 μm wide. The average relative roughness of the cross-section was not measured. The experimental data revealed that the transition region extended in the range of 1000–1500 of the Reynolds number, but this result was obtained by heating the liquid flow, as in the work of Wang and Peng [73].

Pfund et al. [42] measured the pressure drop of water flowing along shallow rectangular microchannels 100–500 μm wide with an aspect ratio (γ) of 0.05. The transition to the turbulence was observed at a Reynolds number equal to 1450. The value of the relative roughness was not measured.

Harms et al. [40] investigated the developing convective heat transfer in rectangular silicon microchannels. Two different channels with an aspect ratio equal to 0.04 and 0.244 were tested and the critical Reynolds number was experimentally determined for the non-isothermal flow.

Mala and Li [44] confirmed, for a water flow through a circular microtube of fused silica and stainless steel, the existence of an early transition from laminar flow to turbulent flow. They explained this fact as the effect of the surface roughness of the microtubes.

Stanley et al. [77] carried out some experiments of liquid flow in rectangular microchannels having a hydraulic diameter from 56 to 260 μm . For water flow he stated that no transition of flow occurred at any size of channels at any Reynolds numbers from 2 to 10 000. The transition occurred for values of Reynolds number between 1500 and 2000 when the hydraulic diameter was larger than 150 μm . For gas flow, the transition was almost completely suppressed below a hydraulic diameter of 80 μm . Varying degrees of suppression were evidenced for hydraulic diameters between 80 and 150 μm .

The experimental analysis of Ding et al. [51] on the pressure drop for R134a and R12 through microchannels with a

triangular and a rectangular cross-section was also devoted to the determination of the range of the critical Reynolds numbers linked to the laminar-to-turbulent transition. They gave a direct measurement of the relative roughness of the tested microchannels. An interesting characteristic of these experimental results is that the role of the relative roughness in anticipating the transition, as underlined by Wu and Little [16], appears not to be confirmed.

Celata et al. [53] tested a capillary tube and found that the transition from laminar to turbulent regime was in accordance with the prediction of the conventional theory for continuum flows; the authors evidenced the role that the high relative roughness could play on the transition.

Li et al. [55] deduced from their experimental results on the friction factors for circular microtubes that the transition from laminar to turbulence in microtubes occurs at Reynolds number equal to 2300; this result is consistent with the well-known result of the conventional theory for continuum flows. In a further work [56] the same authors concluded that the transition from laminar to turbulent occurred at Reynolds numbers between 1700 and 2000.

Yang et al. [57] experienced that the transition occurred when the Reynolds numbers varied from 1200 to 3800 for air, water and R134a through microtubes. The range of critical Reynolds numbers increases with the decrease in tube diameters. The tube size dependency is more sensitive for water flow than for air flow. The authors concluded that, for all the tested fluids, the conventional theory can be adequately used to estimate the pressure drops.

Pfund et al. [58] observed that the laminar to turbulent transition occurred at Reynolds numbers that were lower than the critical Reynolds number for macroscopic ducts (1700–2800); the critical Reynolds number decreased with decreasing channel depth. The flow regime was observed by means of a microscope, a pulsed laser and a CCD video camera; fluorescent dye streams were analysed in order to determine if the flow was laminar or turbulent. They observed that the transition in microchannels was sudden but not discontinuous.

Debray et al. [59] found that the transition from laminar to turbulent regime can be predicted by using the conventional theory for continuum flow. The microchannels tested were rectangular, with a very low aspect ratio. The range of Reynolds numbers in which the transition region drops goes from 2000 to 3000 for an isothermal flow. For a non-isothermal flow the values of the critical Reynolds numbers increase (from 3000 to 16 000).

The tests of Jiang et al. [62] on a non-isothermal single-phase flow of water through a copper micro-heat exchanger revealed that the transition from laminar to turbulent flow occurs much earlier ($Re = 600$) than the predictions of the conventional theory. The fully developed turbulent regime is present for $Re > 2800$. It is particularly important to underline that, when a non-isothermal flow is considered, it is important to specify to which microchannel section Reynolds number considered refers. In fact, since the proper-

ties vary along the channel (specially the fluid viscosity), the Reynolds number can be doubled over the heated length of the microchannels. In addition, in order to partially explain their results, the authors underlined the high relative roughness of the microchannels and the irregular cross-section geometry due to the microfabrication technique used.

Gao et al. [65] measured the friction factor along rectangular microchannels having a fixed width of 25 μm and a height variable between 1000 to 100 μm . The tested microchannels had a low aspect ratio ($0.008 < \gamma < 0.04$). The critical Reynolds numbers observed ranged between 2500 for γ equal to 0.04 and 4000 for γ equal to 0.012. The authors underlined that there is no evidence of a faster transition to turbulence compared to conventional channels.

Qu and Mudawar [69] did not experience early transitions from laminar to turbulent regime for Reynolds numbers ranging between 139 and 1672: only the laminar regime was observed in this range. The same conclusion is valid if one analyses the experimental data of Judy et al. [67] for circular and square microchannels for Reynolds numbers ranging between 0 and 2000.

Hegab et al. [68] found that the transition from laminar to turbulent flow occurs for a Reynolds number between 2000 and 4000 in the case of rectangular microchannels with hydraulic diameters ranging from 112 to 210 μm . They evidenced that the aspect ratio of the channels and the relative roughness did not influence the transition region.

Bucci et al. [70] compared the experimental critical Reynolds numbers obtained for a capillary pipe having a diameter of 290 μm with Preger–Samoilenko equations validated for commercial tubes. They found a poor prediction of these equations, which they explained by observing that the surface roughness of the tested pipe was at the limit of validity of the Preger–Samoilenko equations.

Li et al. [72] observed that for smooth microtubes with a hydraulic diameter ranging between 79.9 and 166.3 μm the transition from laminar to turbulent regime occurs at Reynolds numbers equal to 2000–2300; this fact underlined that the conventional theory for incompressible laminar flow still works for microtubes with diameters larger than 80 μm . They observed that for rough stainless steel microtubes ($D_h = 136.5\text{--}179.8 \mu\text{m}$) having a relative roughness equal to 5% the flow transition occurred at lower Reynolds number ($\approx 1700\text{--}1900$). By using their experimental data, the authors underlined that the conclusion of early transition for fluid flow in rough microtubes cannot be drawn concretely.

In Table 3 are quoted the main experimental results reported in the open literature on the laminar-to-turbulent transition in microchannels.

It is possible to sum up the main results obtained on the value of the critical Reynolds number linked to the laminar-to-turbulent transition in MFD by highlighting the peculiarities with respect to the conventional macrochannels:

Table 3

The experimental data on the laminar-to-turbulent flow transition in microchannels quoted in the open literature

	Cross-section	ε/D_h	γ	Critical Reynolds	Notes <i>IF</i> = isothermal flow <i>NF</i> = nonisothermal flow
Wu and Little [16]	Trap.	Not meas.	0.445	1000–3000	Gas flow; <i>IF</i>
Acosta et al. [17]	Rect.	Not meas.	0.019	2770	Gas flow; <i>IF</i>
		Not meas.	0.033	2770	
Choi et al. [22]	Circ.	Not meas.	0.05	2770	
		Not meas.	–	500–2000	Gas flow; <i>IF</i>
Wang and Peng [73]	Rect.	Not meas.	0.875	300–800	Liquid flow; <i>NF</i>
		Not meas.	0.857	300–800	
Yu et al. [33]	Circ.	Not meas.	0.571	300–800	
		Not meas.	0.286	300–800	
Gui and Scaringe [35]	Trap.	0.015	0.079	1400	Liquid and gas flow; <i>IF</i>
Peng and Peterson [74]	Rect.	Not meas.	0.875	400–1000	Liquid flow; <i>NF</i>
		Not meas.	0.857	400–1000	
Nguyen et al. [76]	Trap.	Not meas.	0.571	400–1000	
		Not meas.	0.286	400–1000	
Peng and Peterson [75]	Rect.	Not meas.	0.292	1000–1500	Liquid flow; <i>NF</i>
		Not meas.	0.75	300–700	Liquid flow; <i>NF</i>
Stanley et al. [77]	Rect.	Not meas.	1	300–700	
		Not meas.	0.5	300–700	
Pfund et al. [42]	Rect.	Not meas.	0.667	200–400	
		Not meas.	0.667	200–400	
Harms et al. [40]	Rect.	Not meas.	1	70–200	
		Not meas.	0.333	70–200	
Mala and Li [44]	Circ.	Not meas.	0.5	70–200	
		Not meas.	–	> 10000	Gas and liquid flow; <i>IF</i>
Ding et al. [51]	Rect.	Not meas.	0.05	1450	Liquid flow; <i>IF</i>
		Not meas.	0.04	1700	Liquid flow; <i>NF</i>
Celata et al. [53]	Circ.	Not meas.	0.244	1510	
		Not meas.	–	300–1500	Liquid flow; <i>IF</i>
Li et al. [55]	Circ.	Not meas.	1	1500–1800	Liquid flow; <i>IF</i>
		Not meas.	0.005	1400–1500	
Li et al. [56]	Circ.	Not meas.	–	1881–2479	Liquid flow; <i>IF</i>
		Not meas.	–	2300	Gas flow; <i>IF</i>
Yang et al. [57]	Circ.	Not meas.	–	1700–2000	Gas flow; <i>IF</i>
		Not meas.	–	1200–3800	Gas and liquid flow; <i>IF</i>
Pfund et al. [58]	Rect.	Not meas.	0.026	1700	Liquid flow; <i>IF</i>
		Not meas.	0.026	1700	Liquid flow; <i>IF</i>
Debray et al. [59]	Rect.	Not meas.	0.019	2000–3000	Liquid flow; <i>IF</i>
		Not meas.	0.009	2000–3000	
Jiang et al. [62]	Rect.	Not meas.	0.005	2000–3000	
		Not meas.	0.005	2000–3000	
Gao et al. [65]	Rect.	Not meas.	0.333	600–2800	Liquid flow; <i>NF</i>
		Not meas.	0.04	2500	Liquid flow; <i>IF</i>
Hegab et al. [68]	Rect.	Not meas.	0.028	2500	
		Not meas.	0.02	3200	
Bucci et al. [70]	Circ.	Not meas.	0.016	3800	
		Not meas.	0.012	4000	
Li et al. [72]	Circ.	Not meas.	0.008	3300	
		Not meas.	0.008	3300	
Li et al. [72]	Circ.	Not meas.	0.99	2000–4000	Liquid flow; <i>IF</i>
		Not meas.	0.905	2000–4000	
Li et al. [72]	Circ.	Not meas.	0.646	2000–4000	
		Not meas.	0.99	2000–4000	
Li et al. [72]	Circ.	Not meas.	–	2800–3200	Liquid flow; <i>IF</i>
		Not meas.	–	2000	Liquid flow; <i>IF</i>

- an earlier laminar-to-turbulent transition with respect to the predictions of the conventional theory has been observed [16,35,44,51,62,73–75];
- the transition is characterized by critical Reynolds numbers larger than the conventional value [77];

- the critical Reynolds numbers depend on the wall roughness in a different way with respect of the large-size channels [16,35,44,53,62,70];
- the critical Reynolds numbers decrease with the microchannel hydraulic diameter [22,57].

2.3. Nusselt number

In Table 4 the experimental results on the Nusselt number in silicon microchannels are summarized and compared.

Wu and Little [78] tested the heat transfer characteristics of nitrogen gas flowing through micro-heat exchangers. The microchannels tested were the same used in [16]. The tests involved both laminar and turbulent flow regimes. The authors found average Nusselt numbers higher than those predicted by the conventional correlations for fully developed laminar flows and for fully developed turbulent flows. They proposed a new correlation in order to predict the average Nusselt numbers in the turbulent regime. In the laminar regime the Nusselt numbers depend on the Reynolds number. The authors evidenced that the very large relative roughness of the microchannels could improve heat transfer coefficients.

By using the electrochemical technique to obtain mass transfer coefficients for rectangular microchannels Acosta et al. [17] concluded that the smooth channel correlations for large-sized channels hold for smooth microchannels in laminar and turbulent regime.

Choi et al. [22] presented experimental data on the Nusselt numbers for Reynolds numbers ranging between 50 and 20 000. Both laminar and turbulent regimes are studied; the Nusselt number depends on the Reynolds number and on the Prandtl number in the laminar regime. They found that in the turbulent regime the Colburn analogy was not valid. The Nusselt number was larger than the prediction of the Colburn analogy. The authors proposed two new correlations for the average Nusselt number in laminar and turbulent regime. The correlations proposed by Choi et al. [22] for microtubes are not in agreement with the correlation proposed by Wu and Little [78].

Rahman and Gui [24] tested the laminar forced convection of water in etched silicon microchannels; they found that the Nusselt numbers were higher than those predicted by analytical solutions for developing laminar flows through rectangular channels.

Yu et al. [33] investigated particularly the heat transfer in microtubes in the turbulent regime ($6000 < Re < 20\,000$); they proposed a correlation for the average Nusselt number. Also in this case, the Nusselt numbers in turbulent regime are larger than those predicted by means of the conventional theory. Peng and Wang [79] investigated the forced convec-

Table 4
Experimental results on the Nusselt number for single-phase internal flows through microchannels

	Nu	D_h [μm]	Cross-section	Test fluids
Wu and Little [78]	↑↑	55.8–72.4	Trap.	N_2
Choi et al. [22]	↑↑	3–81	Circ.	N_2
Yu et al. [33]	↑↑	19–102	Circ.	Water, N_2
Rahman and Gui [24]	↑↑	176–325	Trap.	Water
Peng and Wang [79]	↓↓	646	Rect.	Water
Wang and Peng [73]	↓↓	311–747	Rect.	Water Methanol
Peng et al. [80]	↓↓	311–646	Rect.	Methanol
Peng et al. [31]	↓↓	133–367	Rect.	Water
Peng and Peterson [74]	↓↓	311–747	Rect.	Water Methanol
Peng and Peterson [32]	↓↓	133–367	Rect.	Water
Acosta et al. [17]	≈	369–990	Rect.	He
Harms et al. [40]	≈	404	Rect.	Water
Cuta et al. [81]	↑↑	425	Rect.	R124
Ravigururajan et al. [82]	↑↑	425	Rect.	R124
Nguyen et al. [76]	↑↑	690	Trap.	Water
Adams et al. [83]	↑↑	102–1090	Circ.	Water
Adams et al. [84]	≈	> 1200	quasi Tri.	Water
Tso and Mahulikar [85]	↓↓	717–741	Circ.	Water
Celata et al. [53]	↑↑	130	Circ.	R114
Qu et al. [86]	↓↓	62–169	Trap.	Water
Rahman [87]	↑↑	299–491	Rect.	Water
Debray et al. [59]	↓↓	590–2218	Rect.	Water
Jiang et al. [62]	↑↑	300	Rect.	Water
Kandlikar et al. [64]	↑↑	620–1067	Circ.	Water
Gao et al. [65]	↓↓	199.2–1923	Rect.	Water
Qu and Mudawar [69]	≈	349	Rect.	Water
Warrier et al. [66]	≈	750	Rect.	FC-84
Bucci et al. [70]	≈	290	Circ.	Water
Wu and Cheng [88]	≈	69.2–160	Trap.	Water

↑↑— Nu higher than the conventional theory.

↓↓— Nu lower than the conventional theory.

≈— Nu agrees with the conventional theory.

tion of water in rectangular microchannels ($600 \times 700 \mu\text{m}$). The average convective heat transfer coefficient was determined experimentally. The variation of the Nusselt number with the Reynolds number exhibited an unusual behavior: Nu decreasing with increasing Re in laminar regime. They compared their experimental data with the Sieder–Tate correlation and with the correlation for microchannels proposed by Wu and Little for laminar regime; it was found that the experimental data for low values of the Reynolds number disagree with the correlations used. In general, the experimental Nusselt numbers was lower than the predictions of the correlations used. Wang and Peng [73] conducted experiments on forced convection through stainless steel rectangular microchannels; they employed as test fluids water and methanol. The Nusselt numbers in turbulent regime can be predicted by means of a simple correlation of Reynolds and Prandtl numbers. The correlation proposed is a modification of the Dittus–Boelter equation where the empirical coefficient 0.023 is replaced with 0.00805; that means that in this case the experimental Nusselt numbers were lower than the predictions of the conventional theory. Peng et al. [31] analysed the role of the dimensions of the rectangular microchannels on the Nusselt numbers in laminar and turbulent regimes. They evidenced a strong dependence of the Nusselt number on the aspect ratio of the microchannel and proposed some correlations in which the empirical constants are functions of the microchannel dimensions. These data are inconclusive in order to comprehend the role of the microchannel aspect ratio on the Nusselt number. The Nusselt numbers determined experimentally were lower than the conventional values in laminar and in turbulent regime.

Peng and Peterson [74] investigated the effect on the heat transfer of the variation with the temperature of the fluid thermophysical properties. The experimental data showed that the heat transfer is influenced by the temperature of the liquid, the velocity, Reynolds number and the microchannel aspect ratio. The data demonstrated the existence of an optimum channel size in terms of forced convective flow heat transfer.

Peng et al. [80] conducted an experimental analysis of the influence of liquid velocity, property variations and cross-section configuration on the convective heat transfer behavior. The author underlined that through microchannels there usually exists a sharp liquid temperature rise which causes significant liquid thermophysical property variations with a correlated increase in the Reynolds number along the channel. This fact affects the thermal performance of the microchannel. No comparison with the conventional correlations was carried out.

Peng and Peterson [32] proposed two correlations in order to consider the effect of the aspect ratio of the microchannels (defined as $\gamma = \min[b, a] / \max[b, a]$); the correlations are valid in laminar and in turbulent regime. In laminar and in turbulent regime, the Nusselt number is found to depend on the Reynolds number, the Prandtl number, the microchannel aspect ratio and the ratio between

the hydraulic diameter and the microchannel width (a). The authors concluded that the geometric configuration of the microchannel has a critical effect on the single phase convective heat transfer.

This conclusion is remarked by the same authors in [74]; the geometric parameters, particularly the hydraulic diameter and the aspect ratio (γ) were found to have significant effects on the heat transfer characteristics. It is interesting to note that the post-processing of the same data of Peng et al. [31] effectuated by Obot [10] does not confirm the significant effects of the aspect ratio and of the hydraulic diameter on the Nusselt number for laminar and turbulent flow. The lack of a strong effect of the aspect ratio and of the hydraulic diameter is consistent with the well-established behaviour for macrochannels.

Cuta et al. [81] measured the Nusselt number for the rectangular microchannels of a micro-heat exchanger. The microchannels had a hydraulic diameter of $425 \mu\text{m}$ and R124 was used as test fluid. They found that Nusselt number was larger than the expected values in laminar regime. Also in turbulent regime the Nusselt number increased with the Reynolds number in a different way with respect to the conventional results for large-sized channels (the Nusselt number seems directly proportional to $Re^{0.6}$ instead of $Re^{0.8}$).

The same heat exchanger was tested by Ravigururajan et al. [82]. They studied experimentally the convective heat transfer coefficient inside rectangular microchannels with a height of 1 mm and a width of $270 \mu\text{m}$. R124 was used as test fluid. During the single phase flow experiments the heat transfer coefficient was found 300 to 900% higher than the corresponding values predicted by the conventional theory.

Nguyen et al. [76] proposed a correlation in order to predict the Nusselt number for water flow through trapezoidal microchannels in laminar and in turbulent regime; the Nusselt number is correlated with the Reynolds number by means of an unusual expression. The experimental Nusselt numbers are larger than the predictions of the Dittus–Boelter correlation in turbulent regime.

Adams et al. [83] tested Gnielinski's correlation for the prediction of the Nusselt number in turbulent regime for capillary tubes having a hydraulic diameter between 0.76 and 1.08 mm ($3200 < Re < 23000$) to complement the data provided by Yu et al. [33]. Their experimental results have suggested a modification of Gnielinski's correlation; in fact, the experimental Nusselt numbers were usually higher than those predicted by Gnielinski's correlation. Gnielinski's correlation has been modified by means of a factor F that depends on the Reynolds number and on the hydraulic diameter of the capillary tube.

Successively, Adams et al. [84] conducted further experimental tests on turbulent convective heat transfer through non-circular microchannels with a hydraulic diameter of 1.13 mm. The shape of the cross-section was designed to simulate the interior subchannels of a triangularly arranged rod bundle with rod diameters of 3.2 mm and a pitch to di-

ameter ratio of 1.15. They observed that their experimental Nusselt numbers were well-predicted by Gnielinski's correlation; bearing in mind the results obtained in their previous work [83], they concluded that the hydraulic diameter of 1.2 mm can be assumed as reasonable lower limit for conventional correlations for the Nusselt number.

Harms et al. [40] measured experimentally the thermal resistance as a function of the aspect ratio for the same pressure drop and pumping power. They evidenced that the thermal resistance was smaller for deeper channels. The authors concluded that the predictions of the conventional theory on the Nusselt number continue to be valid for microchannels.

Tso and Mahulikar [85] effectuated an experimental analysis of circular microchannels taking into account the effect of the viscous dissipation by means of the Brinkman number. They found that the experimental data for laminar flow can be correlated well by using the Brinkman number. In their experimental analysis the authors found very small values of the Brinkman number (of the order of 10^{-8}), too low to affect directly the water bulk temperature by means of the viscous dissipation (*primary effect of Brinkman number*). The author underlined that, for the microchannels, the effect of the Brinkman number is linked to the reduction of the dynamic viscosity between the inlet and the outlet of a microchannel due to the increase in the bulk temperature; this fact can reduce the Brinkman number at the exit by about 50% of its inlet value. The authors concluded that the axial variation of the Brinkman number affects the convective heat transfer in microchannels (*secondary effect of Br*).

The authors demonstrated that the secondary effect of the Brinkman number can be used in order to explain the decrease in the Nusselt number when the Reynolds number increases in the laminar regime, as experimentally evidenced by some authors [71,73,78]. In conclusion, since the axial variation of the Brinkman number induces a variation of the velocity and temperature profiles the authors concluded that there is never a fully developed regime in microchannels and that the flow friction models proposed in literature without considering heat transfer must be re-examined for their applicability with heat transfer.

Qu et al. [86] investigated the heat transfer characteristics of water flowing through trapezoidal silicon microchannels with a hydraulic diameter ranging from 62 to 169 μm . The experimental results were compared with the numerical predictions by considering the conjugated heat transfer between the solid and the fluid region. The assumption of constant thermophysical properties and of fully developed flow were used in the numerical analysis. They found that the measured Nusselt numbers were lower than the predicted numerical values. They concluded that the lower Nusselt numbers can be due to the effect of surface roughness of the microchannel walls also in the laminar regime.

Celata et al. [53] compared the experimental Nusselt numbers with the correlation of Hausen for laminar flow

and of Dittus–Boelter, Gnielinski and Adams–Gnielinski for turbulent flow. The authors found that the experimental data were underpredicted by the correlations tested for Reynolds numbers in the range of 100–7000.

Rahman [87] conducted experimental measurements for pressure drop and convective heat transfer in microchannel heat sinks with water as coolant. The measured Nusselt numbers are compared with the conventional forced convection correlations for laminar and turbulent regime. The results showed that the measured values of the average Nusselt number were usually larger than those predicted by the conventional correlations. In order to explain the larger values of the Nusselt numbers the author highlighted the role of the surface roughness on the breakage of the velocity boundary layer.

Debray et al. [59] determined experimentally the convective heat transfer coefficient in a rectangular microchannel for water. The smallest microchannel test evidenced lower Nusselt numbers than those predicted by the conventional Colburn correlation for turbulent flow. The difference seems to augment for higher Reynolds numbers. In laminar regime, the deviations confirmed the behaviour evidenced by Wang and Peng [73]. The authors explained the observed deviations by considering the lack of the wall heat flux uniformity at the walls of microchannel.

Jiang et al. [62] determined experimentally the axial average Nusselt number distribution for rectangular microchannels of two micro heat exchangers. A correlation for the developing Nusselt number was made for water flow as a function of the dimensionless axial length ($X^+ = L/Re/Pr/D_h$). The experimental values are higher than the values predicted by means of Petukhov's correlation for the thermal entry region of circular tubes for the constant heat flux boundary condition.

Kandlikar et al. [64] experienced that the relative roughness affects the Nusselt number; in particular for a 620 μm tube, the heat transfer increased with higher relative roughness. For smooth microtubes their experimental values of local Nusselt number are in good agreement with theoretical correlation in the thermal entry region of a hydrodynamically fully developed flow.

Gao et al. [65] measured the axial distribution of the local Nusselt number inside rectangular microchannels with a fixed width of 25 mm and a height ranging from 1 to 0.1 mm. It was observed that for microchannels having a height larger than 0.4 mm the measured average Nusselt number agreed with the conventional correlations in the laminar regime. For microchannels of a height less than 0.4 mm a significant decrease in the Nusselt number was observed. On the contrary, the friction factor for any microchannel agreed with the conventional theory. The authors underlined that this fact is in contradiction with the Reynolds analogy, which at least predicts variations of the friction factor and of the Nusselt number in the same direction.

Warrier et al. [66] measured the axial distribution of the local Nusselt number for single-phase convection through

rectangular microchannels as a function of the dimensionless axial distance. The experimental results were in good agreement with the prediction of the conventional theory for the developing laminar flow in rectangular ducts.

Qu and Mudawar [69] investigated the heat transfer characteristics of a single-phase micro-heat sink experimentally. The three-dimensional conjugate heat transfer problem was solved numerically. The assumptions of constant solid and fluid properties, excluding water viscosity, were considered in the numerical model. The comparison between the experimental results and the numerical predictions was good.

Bucci et al. [70] found that in the laminar regime the Nusselt number increases with the Reynolds number with an exponent larger than that used in the Hausen correlation. In the turbulent regime, they found that the Gnielinski correlation holds up to a Reynolds number equal to 6000.

Wu and Cheng [88] investigated experimentally 13 different trapezoidal silicon microchannels (25.9–291 μm). They found that the laminar Nusselt number and apparent friction constant increase with the increase of surface roughness and surface hydrophilic property. The Nusselt number increases almost linearly with the Reynolds number at low Reynolds numbers ($Re < 100$).

It is possible to sum up the main results obtained on the value of the Nusselt number for single-phase flow in MFD focusing on the main features, compared to the conventional macrochannels:

- in the laminar regime the Nusselt number increases with the Reynolds number with an exponent ranging from 0.3 to 1.96 [22,24,62,76,78,81,82,88];
- in the laminar regime the Nusselt number decreases when the Reynolds number increases [31,32,59,65,73,79,86];
- in the turbulent regime, the Dittus–Boelter correlation and the Gnielinski correlation have to be corrected for microchannel flows [32,33,73,76,79,82,83];
- the Reynolds analogy fails for microchannels [22,24,31–33,53,59,65];
- the high relative roughness of the walls increases the convective heat transfer in microchannels [64,86–88];
- the variation of viscosity with the temperature affects the heat transfer [80,85].

3. Some remarks about the experimental results

From the experimental data quoted in the open literature on the convective flow through microchannels it is evident that, if the experimental results are used in order to establish the validity of the conventional theory for microchannels, the answer obtained is not univocal. In fact, some authors found that the predictions of the conventional theory agree with the experimental results; on the other hand, for the same range of hydraulic diameter, some found the opposite result. Various reasons have been proposed to explain these

deviations by invoking rarefaction, compressibility, viscous dissipation effects, surface conditions (roughness), property variation with temperature, electro-osmotic effects (Electric Double Layer) and so on. Some authors proposed new correlations in order to predict the friction factors and the Nusselt numbers in microchannels. These correlations are in general based on few experimental points and no theoretical analysis originated them. For these reasons, the reliability of the correlations proposed for microchannels is questionable.

Let us consider the following example; Peng and Peterson [31,32] analysed the friction factor in rectangular microchannels by using water as the working fluid. The experimental friction factors were not in agreement with the theoretical predictions in the laminar regime as in the turbulent one. In particular, they found that in the laminar regime the fully developed Poiseuille number (fRe) was a function of the Reynolds number. Based on these data, Peng et al. [31] proposed the following correlation to predict the friction factor in metallic rectangular microchannels in laminar regime:

$$fRe = \frac{C_{f,l}}{Re^{0.98}} \quad (1)$$

where $C_{f,l}$ is an empirical coefficient; this coefficient depends upon the microchannel geometrical configuration (aspect ratio $\gamma = b/a$).

On the contrary, the conventional theory based on the Navier–Stokes equation for laminar fully developed flow through rectangular microchannels indicates that the friction factor can be calculated as follows:

$$fRe = C(\gamma) \quad (2)$$

where constant C depends on the rectangular aspect ratio (γ) only. The values of C for the three cross-sections shown in Fig. 1 have been calculated by Morini [89].

In Table 6 the values of C and $C_{f,l}$ are quoted as a function of the aspect ratio γ of the microchannels.

The value assumed by the empirical coefficient $C_{f,l}$ are given in [31] for 12 different rectangular microchannels. How $C_{f,l}$ depends on the microchannel aspect ratio γ is not clear. As it is possible to note in Table 6, for $\gamma = 0.5$ the authors found two different values of $C_{f,l}$ (28 600 and 5200); the same occurs for $\gamma = 1$ (109 000 and 32 400). The authors concluded that $C_{f,l}$ has to be considered as a function of other geometrical parameter; they proposed to link $C_{f,l}$ with the ratio between the hydraulic diameter of the channel and the center-to-center distance between two adjacent microchannels. This conclusion is unexpected if one considers that the friction factor for a channel cannot depend on the near channels.

In Fig. 2 the experimental data of Peng and Peterson [32] (solid symbols) are quoted and compared with the predictions of Eq. (1) (dashed line). Obviously, the agreement between the experimental data of Peng and Peterson [32] and the correlation of Peng et al. [31] is good; in this comparison two rectangular microchannels having a hydraulic diameter

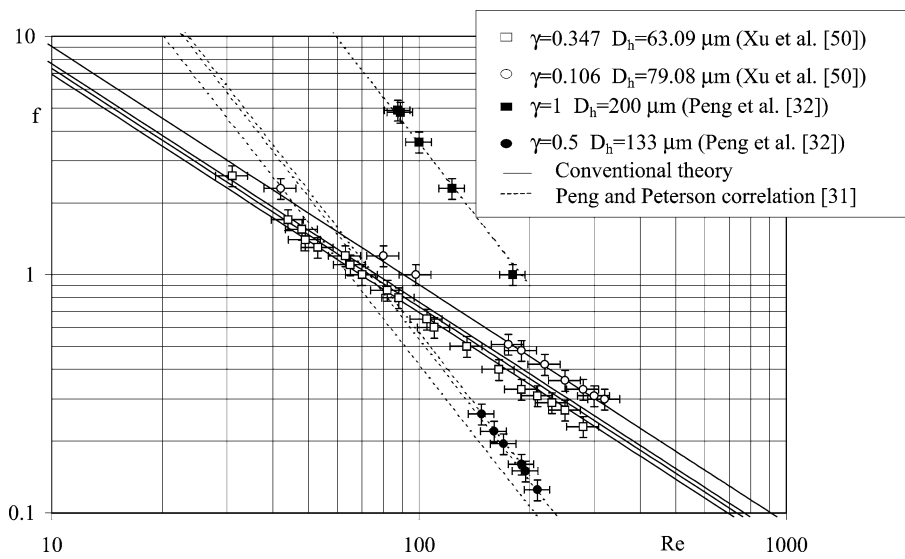


Fig. 2. Comparison between the experimental results of Xu et al. [50] and the correlation of Peng et al. [31].

of 133 and 200 μm are considered. The value of the friction factor predicted by the conventional theory is quoted in Fig. 2 with a solid line. It is interesting to note that the friction factor measured by Peng and Peterson is higher than the expected value when the Reynolds number is low. In Fig. 2 are quoted the experimental data on the friction factor obtained by Xu et al. [50] for silicon rectangular microchannels having a smaller hydraulic diameter than those used by Peng and Peterson [32]. For smaller microchannels, the deviation of the conventional theory should be greater. On the contrary, it is possible to note that the results of Xu et al. [50] are in good agreement with the predictions of the classical theory and the correlation of Peng et al. does not hold for these data. In order to explain this different behaviour it is important to underline that the microchannels tested by Xu et al. were made in silicon wafers by means of a photolithographic technique and the microchannels tested by Peng and Peterson were made in a stainless steel plate by means of micro-machining. For this reason the microchannels used by Peng and Peterson could have a higher wall relative roughness with respect to those of Xu et al. (the relative roughness of the microchannels tested by Peng and Peterson is unknown). The relative roughness could be important also in the laminar regime for microchannels because of the large surface-to-volume ratio typical of these systems. This could be a “scaling effect” linked to the smaller dimensions of the microchannels.

Another possible explanation for this difference can be given by observing the uncertainty in measuring of the Poiseuille number (fRe). It can be demonstrated that the diameter measurement is the most critical parameter to the overall fRe uncertainty measurement. The diameters of these small channels have to be characterized with SEM and the very best uncertainty for these instruments is about 1%. More realistically, this uncertainty will be 2–3% contributing alone to an 8–12% uncertainty in fRe [11]. If appropriate

calibration is not performed, this uncertainty can be much greater and this fact can explain some anomalous deviations.

In addition, in several experimental works, the incorrect evaluation of the manifolds pressure drop could be responsible of the deviation of the experimental friction factor from the classical value of laminar theory. All of the friction factor measurements were conducted in a similar fashion. The microchannels are connected with two manifolds (inlet and outlet) and the measured pressure differential is the potential between the two reservoirs. Minor losses due to the inlet, outlet and hydrodynamic development are considered into the measured total pressure drop.

The data of Harms et al. [40] can be used to test the influence of this point. The experimental data of Harms et al. on the total pressure drop (channel + inlet and outlet manifolds pressure drop) for laminar flow through rectangular microchannels as a function of the flow rate make it possible to observe that the agreement with the conventional theory is good for lower flow rates.

This fact underlines the role of the manifold pressure drop on the total pressure drop. The authors evidenced that, at low flow rates, the channel core dominates the flow friction ($\Delta p_{\text{ch}}/\Delta p \approx 93\%$) and the experimental results follow the laminar theory. At a high flow rate the manifold pressure drop is significant and the deviation of the theoretical prediction augments.

It is interesting to observe that in the experimental works where the inlet and outlet manifold pressure drops are considered, the conventional correlations are used in order to calculate these minor losses. This fact underlines that many authors used the “implicit hypothesis” that the minor losses in microchannels have to be well predicted by means of the conventional theory!

In addition, in the paper of Harms et al. [40] the authors correctly used the apparent Poiseuille number instead of the fully developed Poiseuille number by emphasizing the

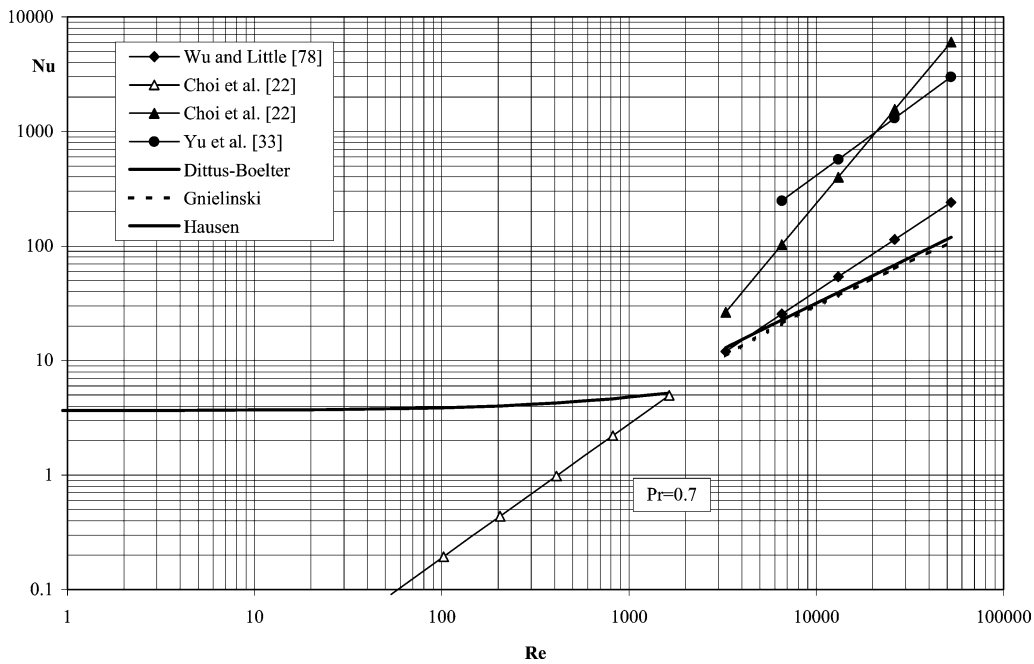


Fig. 3. Comparison between the experimental correlations for the Nusselt number in microchannels: Gas flow.

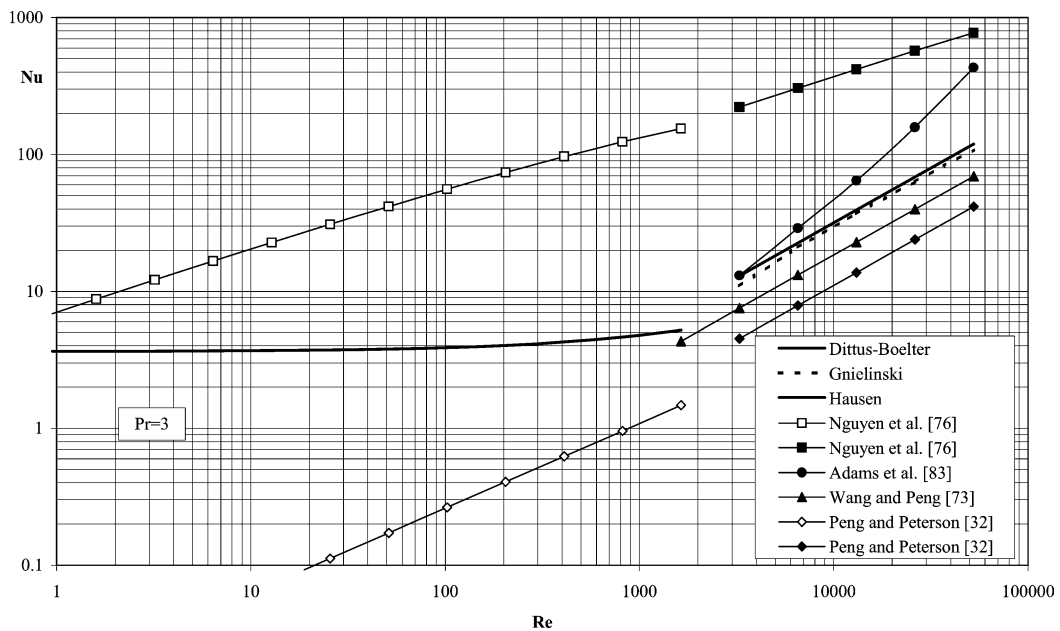


Fig. 4. Comparison between the experimental correlations for the Nusselt number in microchannels: Liquid flow.

important role of the developing flow in the entrance region of microchannels. These effects have been considered negligible in many other works.

In Figs. 3 and 4 a comparison of the main correlations proposed to calculate the Nusselt number for microchannels is shown for laminar and turbulent regime. For gas flow ($Pr = 0.7$) the correlations proposed by Wu and Little [78], Choi et al. [22] and Yu et al. [33] are considered in Fig. 3.

These correlations are quoted in Table 5. In Fig. 3, the Hausen correlation [90] valid for a laminar flow through a

isothermal tube, the Dittus–Boelter correlation [90] and the Gnielinski correlation [90] for turbulent flows are shown.

In the laminar regime, the correlation proposed by Choi et al. [22] predicts a lower Nusselt number than the Hausen correlation. It has been evidenced by several authors that the Nusselt number increases with the Reynolds number more quickly than the prediction of the conventional theory in the laminar regime.

In the turbulent regime, the correlations of Wu and Little [78], Choi et al. [22] and Yu et al. [33] are not in agreement

Table 5
Summary of correlations for laminar convective heat transfer inside microchannels

	Cross-section	Correlation
Wu and Little [78]	Trap.	$Nu = 0.00222Pr^{0.4}Re^{1.08}$ for $Re > 3000$
Choi et al. [22]	Circ.	$Nu = 0.000972Re^{1.17}Pr^{1/3}$ for $Re < 2000$ $Nu = 3.82 \times 10^{-6}Re^{1.96}Pr^{1/3}$ for $2500 < Re < 20000$
Yu et al. [33]	Circ.	$Nu = 0.007Re^{1.2}Pr^{0.2}$ for $6000 < Re < 20000$
Wang and Peng [73]	Rect.	$Nu = 0.00805Re^{0.8}Pr^{1/3}$ for $Re > 1500$
Peng and Peterson [32]	Rect. ¹	$Nu = 0.1165(D_h/W_c)^{0.81}\gamma^{0.79}Re^{0.62}Pr^{1/3}$ Laminar regime ¹ $Nu = 0.072(D_h/W_c)^{1.15}(1 - 2.421(\gamma - 0.5)^2)Re^{0.8}Pr^{1/3}$ Turbulent regime
Nguyen et al. [76]	Trap.	$Nu = 8.39Re^{0.5} - 1.33Re^{2/3}$ Laminar regime $Nu = 4.73Re^{0.5} - 0.22Re^{2/3}$ Turbulent regime
Adams et al. [83]	Circ. ^{2,3,4}	$Nu = Nu_{Gn}(1 + F)$ Turbulent regime
Jiang et al. [62]	Rect. ⁵	$Nu = 0.52(X^+)^{-0.62}$ for $X^+ < 0.05$ $Nu = 2.02(X^+)^{-0.31}$ for $X^+ > 0.05$
Wu and Cheng [88]	Trap. ⁶	$Nu = C_1Re^{0.946}Pr^{0.488}(1 - c/a)^{3.547}(\epsilon/D_h)^{0.041}\gamma^{-3.577}(D_h/L)^{1.369}$ for $10 < Re < 100$ $Nu = C_2Re^{0.148}Pr^{0.163}(1 - c/a)^{0.908}(\epsilon/D_h)^{0.033}\gamma^{-1.001}(D_h/L)^{0.798}$ for $100 < Re < 1500$

¹ W_c = center-to-center distance between two adjacent microchannels.

² $F = 7.610^{-5}Re(1 - (D/1.164)^2)D$ in mm.

³ $f = (1.82 \log Re - 1.64)^{-2}$.

⁴ $Nu_{Gn} = \frac{(f/8)(Re-1000)Pr}{1+12.7(f/8)^{0.5}(Pr^{2/3}-1)}$.

⁵ $X^+ = RePrD_h/L$.

⁶ $C_1 = 6.7$ for silicon surfaces $C_1 = 6.6$ for thermal oxide surface $C_2 = 47.8$ for silicon surfaces $C_2 = 54.4$ for thermal oxide surface.

Table 6
Values of C and $C_{f,l}$ defined in Eqs. (1), (2) as a function of the microchannel cross-section geometry

γ	Microchannel cross-section					
	Rectangular		Trapezoidal		Double-trapezoidal	
	C [89]	$C_{f,l}$ [31]	γ	C [89]	γ	C [89]
0	24	–	0	24	0	24
0.01	23.677	–	0.01	23.597	0.1	21.507
0.05	22.477	–	0.047	22.174	0.2	19.501
0.1	21.169	–	0.088	20.737	0.4	16.746
0.2	19.071	–	0.156	18.650	0.5	15.923
0.3	17.512	–	0.211	17.244	0.6	15.403
0.333		24200				
0.4	16.368	–	0.293	15.565	0.7	15.126
0.5	15.548	28600	0.352	14.690	0.8	15.027
		5200				
0.6	14.980	–	0.414	14.063	0.9	15.043
0.667		42600				
0.7	14.605	–	0.522	13.654	1	15.111
0.75		44800				
0.8	14.378	–	0.583	13.679	1.2	15.081
0.9	14.261	–	0.660	13.622	1.3	14.785
1	14.227	109000	0.707	13.308	1.414	14.055
		32400				

with each other. In particular, it is well evident that in the turbulent regime the correlations of Choi et al., Yu et al. and Wu and Little predict higher Nusselt numbers than the Dittus–Boelter and the Gnielinski correlations.

In Fig. 4 the correlations of Nguyen et al. [76], Adams et al. [83], Wang and Peng [73] and Peng and Peterson [32] are compared with the Hausen, Dittus–Boelter and Gnielinski correlations [90] for a liquid with $Pr = 3$. In the laminar regime the correlations proposed by Peng and Peterson [32] and Nguyen et al. [76] are in disagreement with each other and with the Hausen correlation.

In the turbulent regime, the correlations of Wang and Peng [73] and Peng and Peterson [32] give lower Nusselt numbers than the Dittus–Boelter and the Gnielinski correlations. On the contrary, the opposite result is obtained if the correlations proposed by Nguyen et al. and Adams et al. are used.

4. Conclusions

By comparing the available experimental data on single-phase convective heat transfer through microchannels, it is evident that further systematic studies are required to generate a sufficient body of knowledge of the transport mechanism responsible for the variation of the flow structure and heat transfer in microchannels. From an chronological analysis of the experimental results quoted in this paper it is possible to extrapolate how the deviations between the behaviour of fluids through microchannels with respect to the large-sized channels are decreasing. This fact can be explained by taking into account the dramatic improvement of the techniques of microfabrication with the consequent reduction of the issues of surface roughness of the microchannels and a more appropriate control of the channel cross-sections; for this reason the results of the older studies may not provide useful comparisons. Another possible explanation of the decrease of the observed deviations is related to the increase of the reliability/accuracy of the more recent experimental data quoted in literature. Anyhow, the analysis conducted in this review confirmed that the understanding of the fluid flow and the heat transfer mechanisms in microchannels has to be considered, at the moment, a scientific open question.

Acknowledgement

The author is grateful to Prof. M. Spiga, Prof. M.W. Collins and Prof. S. Piva for helpful discussions on microchannel heat transfer.

References

- [1] T.A. Ameel, R.O. Warrington, R.S. Wegeng, M.K. Drost, Miniaturization technologies applied to energy systems, *Energy Convers. Mgmt.* 38 (1997) 969–982.
- [2] M. Gad-el-Hak (Ed.), *The MEMS Handbook*, CRC Press, New York, 2001.
- [3] N.T. Nguyen, Micromachined flow sensors—A review, *Flow Meas. Instrum.* 8 (1997) 7–16.
- [4] A.R. Abramson, C.L. Tien, Recent developments in microscale thermophysical engineering, *Microscale Thermophys. Engrg.* 3 (1999) 229–244.
- [5] A.B. Duncan, G.P. Peterson, Review of microscale heat transfer, *ASME Appl. Mech. Rev.* 47 (1994) 397–428.
- [6] X.F. Peng, B.X. Wang, Forced convection and boiling characteristics in microchannels, in: *Proceedings of 11th Int. Heat Transfer Conference*, Kyongyu, Korea, vol. 1, 1998, pp. 371–390.
- [7] D.K. Bailey, T.A. Ameel, R.O. Warrington, T.I. Savoie, Single phase forced convection heat transfer in microgeometries—A review, in: *IECEC Conference ASME-FL*, Orlando, USA, 1995, ES-396.
- [8] B. Palm, Heat transfer in microchannels, *Microscale Thermophys. Engrg.* 5 (2001) 155–175.
- [9] C.B. Sobhan, S.V. Garimella, A comparative analysis of studies on heat transfer and fluid flow in microchannels, *Microscale Thermophys. Engrg.* 5 (2001) 293–311.
- [10] N.T. Obot, Toward a better understanding of friction and heat/mass transfer in microchannels—A literature review, *Microscale Thermophys. Engrg.* 6 (2002) 155–173.
- [11] W.J. Bowman, D. Maynes, A review of micro-heat exchanger flow physics, fabrication methods and application, in: *Proceedings of ASME IMECE 2001*, New York, USA, 2001, HTD-24280.
- [12] A.A. Rostami, N. Saniei, A.S. Mujumdar, Liquid flow and heat transfer in microchannels: A review, *Heat Technol.* 18 (2000) 59–68.
- [13] A.A. Rostami, A.S. Mujumdar, N. Saniei, Flow and heat transfer for gas flowing in microchannels: A review, *Heat Mass Transfer* 38 (2002) 359–367.
- [14] S.G. Kandlikar, W. Grande, Evolution of microchannel flow passages—thermo-hydraulic performance and fabrication technology, *Heat Transfer Engrg.* 24 (2003) 3–17.
- [15] G.M. Whitesides, A.D. Stroock, Flexible methods for microfluidics, *Phys. Today* 6 (2001) 42–48.
- [16] P. Wu, W.A. Little, Measurement of friction factors for the flow of gases in very fine channels used for microminiature Joule–Thompson refrigerators, *Cryogenics* 23 (1983) 273–277.
- [17] R.E. Acosta, R.H. Muller, W.C. Tobias, Transport processes in narrow (Capillary) channels, *AIChE J.* 31 (1985) 473–482.
- [18] J. Harley, H.H. Bau, Fluid flow in micron and sub-micron size channels, in: *Proceedings of IEEE, MEMS*, 1989, pp. 25–28.
- [19] J. Pfaller, J. Harley, H.H. Bau, J.N. Zemel, Liquid transport in micron and submicron channels, *Sensors Actuators A* 21–23 (1990) 431–434.
- [20] J. Pfaller, J. Harley, H.H. Bau, J.N. Zemel, Liquid and gas transport in small channels, in: *Proceedings of ASME DSC*, vol. 31, 1990, pp. 149–157.
- [21] J. Pfaller, J. Harley, H.H. Bau, J.N. Zemel, Gas and liquid flow in small channels, in: *Micromechanical Sensors, Actuator and Systems*, ASME DSC, vol. 32, Atlanta, GA, 1991, pp. 49–60.
- [22] S.B. Choi, R.F. Barron, R.O. Warrington, Fluid flow and heat transfer in microtubes, in: *Micromechanical Sensors, Actuators and Systems*, ASME DSC, vol. 32, Atlanta, GA, 1991, pp. 123–134.
- [23] W. Urbanek, J.N. Zemel, H.H. Bau, An investigation of the temperature dependence of Poiseuille numbers in microchannel flow, *J. Microtech. Microengng.* 3 (1993) 206–209.
- [24] M.M. Rahman, F.J. Gui, Experimental measurements of fluid flow and heat transfer in microchannel cooling passages in a chip substrate, in: *Advances in Electronic Packaging*, in: ASME EEP, vol. 199, 1993, pp. 685–692.
- [25] M.M. Rahman, J.F. Gui, Design fabrication and testing of microchannel heat sink for aircraft avionics cooling, in: *Proceedings of the 28th Intersociety Energy Conversion Eng. Conf.*, vol. 1, 1993, pp. 1–6.
- [26] P. Wilding, J. Pfaller, J.N. Zemel, H.H. Bau, L.J. Kricka, Manipulation and flow of biological fluids in straight channels micromachined in silicon, *Clin. Chem.* 40 (1994) 43–47.

- [27] E.B. Arklic, K.S. Breuer, M.A. Schmidt, Gaseous flow in microchannels, in: *Application of Microfabrication to Fluid Mechanics*, in: ASME FED, vol. 197, 1994, pp. 57–66.
- [28] K. Pong, C. Ho, J. Liu, Y. Tai, Non-linear pressure distribution in uniform microchannels, in: *Application of Microfabrication to Fluid Mechanics*, in: ASME FED, vol. 197, 1994, pp. 51–56.
- [29] J. Liu, Y.C. Tai, C.M. Ho, MEMS for pressure distribution studies of gaseous flows through uniform microchannels, in: *Proceedings of 8th Ann. Int. Workshop MEMS, IEEE*, 1995, pp. 209–215.
- [30] J.C. Shih, C.M. Ho, J. Liu, Y.C. Tai, Monatomic and polyatomic gas flow through uniform microchannels, *ASME DSC 59 (1996)* 197–203.
- [31] X.F. Peng, G.P. Peterson, B.X. Wang, Frictional flow characteristics of water flowing through rectangular microchannels, *J. Exp. Heat Transfer 7 (1995)* 249–264.
- [32] X.F. Peng, G.P. Peterson, Convective heat transfer and flow friction for water flow in microchannel structures, *Internat. J. Heat Mass Transfer 39 (1996)* 2599–2608.
- [33] D. Yu, R.O. Warrington, R. Barron, T. Ameel, An experimental and theoretical investigation of fluid flow and heat transfer in microtubes, in: *Proceedings of ASME/JSME Thermal Engineering Joint Conf., Maui, HI, 1995*, pp. 523–530.
- [34] J. Harley, Y. Huang, H.H. Bau, J.N. Zemel, Gas flow in microchannels, *J. Fluid Mech. 284 (1995)* 257–274.
- [35] F.J. Gui, R.P. Scaringe, Enhanced heat transfer in the entrance region of microchannels, in: *Proceedings of the 30th Intersociety Energy Conversion Eng. Conf., vol. 2, 1995*, pp. 289–294.
- [36] X.N. Jiang, Z.Y. Zhou, J. Yao, Y. Li, X.Y. Ye, Micro-fluid flow in microchannel, in: *Proceedings of Transducers '95, Stockholm, Sweden, 1995*, pp. 317–320.
- [37] X.N. Jiang, Z.Y. Zhou, X.Y. Huang, C.Y. Liu, Laminar flow through microchannels used for microscale cooling systems, in: *Proceedings of 97 IEEE/CPMT Electronic Packaging Technology Conference, 1997*, pp. 119–122.
- [38] M. Richter, P. Woias, D. Weiß, Microchannels for applications in liquid dosing and flow-rate measurements, *Sensors Actuators A 62 (1997)* 480–483.
- [39] T.M. Harms, M.J. Kazmierczak, F.M. Gerner, A. Holke, H.T. Henderson, J. Pilchowski, K. Baker, Experimental investigation of heat transfer and pressure drop through deep microchannels in a (110) silicon substrate, in: *Proceedings of ASME Heat Transfer Division*, in: ASME HTD, vol. 351-1, 1997, pp. 347–357.
- [40] T.M. Harms, M.J. Kazmierczak, F.M. Gerner, Developing convective heat transfer in deep rectangular microchannels, *Internat. J. Heat Fluid Flow 20 (1999)* 149–157.
- [41] R.L. Webb, M. Zhang, Heat transfer and friction in small diameters channels, *Microscale Termophys. Engrg. 2 (1998)* 189–202.
- [42] D.A. Pfund, A. Shekarriz, A. Popescu, J.R. Welty, Pressure drops measurements in microchannels, in: *Proceedings of MEMS*, in: ASME DSC, vol. 66, 1998, pp. 193–198.
- [43] S.M. Flockhart, R.S. Dhariwal, Experimental and numerical investigation into the flow characteristics of channels etched in (100) silicon, *J. Fluid Engrg. 120 (1998)* 291–295.
- [44] G.M. Mala, D. Li, Flow characteristics of water in microtubes, *Internat. J. Heat Fluid Flow 20 (1999)* 142–148.
- [45] I. Papautsky, J. Brazzle, T. Ameel, A.B. Frazier, Laminar fluid behavior in microchannels using micropolar fluid theory, *Sensors Actuator A 73 (1999)* 101–108.
- [46] C.D. Meinhardt, S.T. Wereley, J.G. Santiago, PIV measurements of a microchannel flow, *Experiments Fluids 27 (1999)* 414–419.
- [47] B. Xu, K.T. Ooi, N.T. Wong, C.Y. Liu, W.K. Choi, Liquid flow in microchannels, in: *Proceedings of the 5th ASME/JSME Joint Thermal Engrg. Conference, San Diego, CA, 1999*, pp. 150–158.
- [48] W. Qu, M. Mala, D. Li, Pressure-driven water flows in trapezoidal silicon microchannels, *Internat. J. Heat Mass Transfer 43 (2000)* 353–364.
- [49] K.V. Sharp, R.J. Adrian, D.J. Beebe, Anomalous transition to turbulence in microtubes, in: *Proceedings of Int. Mech. Eng. Cong. Expo., 5th Micro-Fluidic Symp., Orlando, FL, 2000*, pp. 150–158.
- [50] B. Xu, K.T. Ooi, N.T. Wong, W.K. Choi, Experimental investigation of flow friction for liquid flow in microchannels, *Internat. Comm. Heat Mass Transfer 27 (2000)* 1165–1176.
- [51] L.S. Ding, H. Sun, X.L. Sheng, B.D. Lee, Measurement of friction factors for R134a and R12 through microchannels, in: *Proceedings of Symposium on Energy Engrg. in the 21st Century, vol. 2, 2000*, pp. 650–657.
- [52] T. Araki, K.M. Soo, I. Hiroshi, S. Kenjiro, An experimental investigation of gaseous flow characteristics in microchannels, in: G.P. Celata, et al. (Eds.), *Proceedings of International Conference on Heat Transfer and Transport Phenomena in Microscale*, Begell House, New York, USA, 2000, pp. 155–161.
- [53] G.P. Celata, M. Cumo, M. Guglielmi, G. Zummo, Experimental investigation of hydraulic and single phase heat transfer in 0.130 mm capillary tube, in: G.P. Celata, et al. (Eds.), *Proceedings of International Conference on Heat Transfer and Transport Phenomena in Microscale*, Begell House, New York, USA, 2000, pp. 108–113.
- [54] J. Judy, D. Maynes, B.W. Webb, Liquid flow pressure drop in microtubes, in: G.P. Celata, et al. (Eds.), *Proceedings of International Conference on Heat Transfer and Transport Phenomena in Microscale*, Begell House, New York, USA, 2000, pp. 149–154.
- [55] Z.X. Li, D.X. Du, Z.Y. Guo, Characteristics of frictional resistance for gas flow in microtubes, in: *Proceedings of Symposium on Energy Engrg. in the 21st Century, vol. 2, 2000*, pp. 658–664.
- [56] Z.X. Li, D.X. Du, Z.Y. Guo, Experimental study on flow characteristics of liquid in circular microtubes, in: G.P. Celata, et al. (Eds.), *Proceedings of International Conference on Heat Transfer and Transport Phenomena in Microscale*, Begell House, New York, USA, 2000, pp. 162–168.
- [57] C.Y. Yang, H.T. Chien, S.R. Lu, R.J. Shyu, Friction characteristics of water, R134a and air in small tubes, in: G.P. Celata, et al. (Eds.), *Proceedings of International Conference on Heat Transfer and Transport Phenomena in Microscale*, Begell House, New York, USA, 2000, pp. 168–174.
- [58] D. Pfund, D. Rector, A. Shekarriz, A. Popescu, J. Welty, Pressure drop measurements in a microchannel, *AIChE J. 46 (2000)* 1496–1507.
- [59] F. Debray, J.P. Franc, T. Maitre, S. Reynaud, Mesure des coefficient de transfert thermique par convection forcée en mini-canaux, *Mec. Ind. 2 (2001)* 443–454.
- [60] P. Lalonde, S. Colin, R. Caen, Mesure de debit de gaz dans les microsystemes, *Mec. Ind. 2 (2001)* 355–362.
- [61] S.E. Turner, H. Sun, M. Faghri, O.J. Gregory, Compressible gas flow through smooth and rough microchannels, in: *Proceedings of IMECE 2001, New York, USA, 2001*, HTD-24145.
- [62] P.X. Jiang, M.H. Fan, G.S. Si, Z.P. Ren, Thermal-hydraulic performance of small scale micro-channel and porous-media heat-exchangers, *Internat. J. Heat Mass Transfer 44 (2001)* 1039–1051.
- [63] L. Ren, W. Qu, D. Li, Interfacial electrokinetic effects on liquid flow in microchannels, *Internat. J. Heat Mass Transfer 44 (2001)* 3125–3134.
- [64] S.G. Kandlikar, S. Joshi, S. Tian, Effect of channel roughness on heat transfer and fluid flow characteristics at low Reynolds numbers in small diameter tubes, in: *Proc. of 35th National Heat Transfer Conference, Anaheim CA, USA, 2001*, paper 12134.
- [65] P. Gao, S. Le Person, M. Favre-Marinet, Scale effects on hydrodynamics and heat transfer in two-dimensional mini and microchannels, *Internat. J. Thermal Sci. 41 (2002)* 1017–1027.
- [66] G.R. Warrier, V.K. Dhir, L.A. Momoda, Heat transfer and pressure drop in narrow rectangular channels, *Exp. Thermal Fluid Sci. 26 (2002)* 53–64.
- [67] J. Judy, D. Maynes, B.W. Webb, Characterization of frictional pressure drop for liquid flows through microchannels, *Internat. J. Heat Mass Transfer 45 (2002)* 3477–3489.
- [68] H.E. Hegab, A. Bari, T. Ameel, Friction and convection studies of R134a in microchannels within the transition and turbulent flow regimes, *Exp. Heat Transfer 15 (2002)* 245–259.

- [69] W. Qu, I. Mudawar, Experimental and numerical study of pressure drop and heat transfer in a single-phase micro-channel heat sink, *Internat. J. Heat Mass Transfer* 45 (2002) 2549–2565.
- [70] A. Bucci, G.P. Celata, M. Cumo, E. Serra, G. Zummo, Fluid flow and single-phase flow heat transfer of water in capillary tubes, in: *Proceedings of the Int. Conference on Minichannels and Microchannels*, Rochester, USA, 2003, paper ICMM-1037.
- [71] H.Y. Wu, P. Cheng, Friction factors in smooth trapezoidal silicon microchannels with different aspect ratios, *Internat. J. Heat Mass Transfer* 46 (2003) 2519–2525.
- [72] Z.X. Li, D.X. Du, Z.Y. Guo, Experimental study on flow characteristics of liquid in circular microtubes, *Microscale Thermophys. Engrg.* 7 (2003) 253–265.
- [73] B.X. Wang, X.F. Peng, Experimental investigation on liquid forced-convection heat transfer through microchannels, *Internat. J. Heat Mass Transfer Suppl.* 37 (1) (1994) 73–82.
- [74] X.F. Peng, G.P. Peterson, The effect of thermo-fluid and geometrical parameters on convection of liquids through rectangular microchannels, *Internat. J. Heat Mass Transfer* 38 (1995) 755–758.
- [75] X.F. Peng, G.P. Peterson, Forced convection heat transfer of single-phase binary mixtures through microchannels, *Exp. Thermal Fluid Sci.* 12 (1996) 98–104.
- [76] N.T. Nguyen, D. Bochnia, R. Kiehnscherrf, W. Dözel, Investigation of forced convection in microfluid systems, *Sensors Actuators A* 55 (1996) 49–55.
- [77] R.S. Stanley, R.F. Barron, T.A. Ameel, Two-phase flow in microchannels, in: *Proceedings of Micro Electro Mechanical Systems (MEMS)*, in: *DSC/ASME*, vol. 62, 1997, pp. 143–152.
- [78] P. Wu, W.A. Little, Measurement of the heat transfer characteristics of gas flow in fine channel heat exchangers used for microminiature refrigerators, *Cryogenics* 24 (1984) 415–420.
- [79] X.F. Peng, B.X. Wang, Forced convection and flow boiling heat transfer for liquid flowing through microchannels, *Internat. J. Heat Mass Transfer* 36 (1993) 3421–3427.
- [80] X.F. Peng, B.X. Wang, G.P. Peterson, H.B. Ma, Experimental investigation of heat transfer in flat plates with rectangular microchannels, *Internat. J. Heat Mass Transfer* 38 (1995) 127–137.
- [81] J.M. Cuta, C.E. McDonald, A. Shekarriz, Forced convection heat transfer in parallel channel array microchannel heat exchanger, in: *Advances in Energy Efficiency, Heat/Mass Transfer Enhancement*, in: *ASME-PID, 2/HTD*, vol. 338, 1996, pp. 17–23.
- [82] T.S. Ravigururajan, J.M. Cuta, C.E. McDonald, M.K. Dorst, Single-phase thermal performance characteristics of a parallel micro-channel heat exchanger, in: *Proc. of US National Heat Transfer Conference*, in: *ASME-HTD*, vol. 329, 1996, pp. 157–166.
- [83] T.M. Adams, S.I. Abdel-Khalik, S.M. Jeter, Z.H. Qureshi, An Experimental investigation of single-phase forced convection in microchannels, *Internat. J. Heat Mass Transfer* 41 (1998) 851–857.
- [84] T.M. Adams, M.F. Dowling, S.I. Abdel-Khalik, S.M. Jeter, Applicability of traditional turbulent single phase forced convection correlations to non-circular microchannels, *Internat. J. Heat Mass Transfer* 42 (1999) 4411–4415.
- [85] C.P. Tso, S.P. Mahulikar, Experimental verification of the role of Brinkman number in microchannels using local parameters, *Internat. J. Heat Mass Transfer* 43 (2000) 1837–1849.
- [86] W. Qu, G.M. Mala, D. Li, Heat transfer for water flow in trapezoidal silicon microchannels, *Internat. J. Heat Mass Transfer* 43 (2000) 3925–3936.
- [87] M.M. Rahman, Measurements of heat transfer in microchannel heat sinks, *Internat. Comm. Heat Mass Transfer* 27 (2000) 495–506.
- [88] H.Y. Wu, P. Cheng, An experimental study of convective heat transfer in silicon microchannels with different surface conditions, *Internat. J. Heat Mass Transfer* 46 (2003) 2547–2556.
- [89] G.L. Morini, Laminar-to-turbulent flow transition in microchannels, *Microscale Thermophys. Engrg.* 8 (2004) 15–30.
- [90] M.N. Ozisik, *Heat Transfer: A Basic Approach*, McGraw-Hill, New York, 1985.